Precalculus Lecture Notes

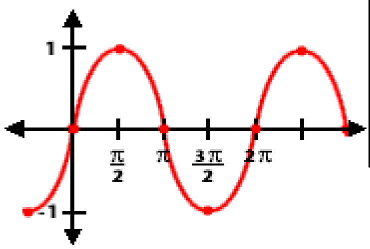
Introduction to the Graphs of Sine and Cosine Functions

>> Key Concepts:

- The graph of the sine function oscillates and repeats without end. The graph of the cosine function is the same as that of the sine function shifted to the left by $\pi/2$ units.
- The **amplitude** of an oscillating function is half the difference between the maximum and minimum values of the function.
- The **period** of an oscillating function is the length of the interval over which the function goes through one full cycle before it repeats.

Graphing y = sin x

independent variable



y = sin x		
x (radians)	x (degrees)	sin x
0	0°	0
$\pi/6$	30°	1/2
$\pi/4$	45°	$\sqrt{2}/2$
$\pi/3$	60°	$\sqrt{3}/2$
$\pi/2$	90°	1
π	180°	0
$3\pi/2$	270°	-1
2π	360°	0

You can graph the function y = sin x by making a table of values and plotting the points.

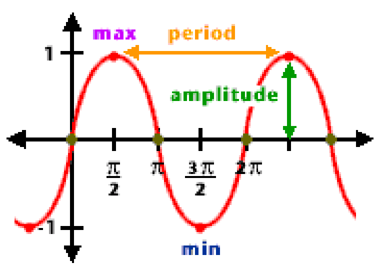
The graph of the sine function is pictured to the left. It repeats itself without stopping.

NOTE: x is given in **radians**, not degrees.

amplitude: half of the difference from the minimum height to the maximum height.
period: length required for a function to produce one full cycle before it begins to repeat its cycles.
zeros: where the curve crosses the x-axis.

Graphing y = sin x

max = 1
min = -1
amplitude = 1
period = 2π



max's occur at $\frac{\pi}{2}$ ± any multiple of 2π.
min's occur at $\frac{3\pi}{2}$ ± any multiple of 2π.
zeros occur at any multiple of π.

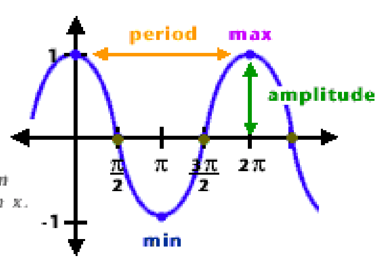
There are some special terms that describe the graphs of oscillating functions like y = sin x:

1. The **amplitude** is equal to half the difference between the maximum height and the minimum height. For y = sin x, the amplitude is $(1/2)[1 - (-1)] = 1$. You can think of the amplitude as the vertical distance that the function covers.
2. The **period** is the length of the largest interval the function covers without repeating. The sine function repeats every 2π radians.

Since the sine function repeats every 2π radians, its **maxima**, **minima**, and **zeros** also repeat themselves regularly.

Graphing y = cos x

max = 1
min = -1
amplitude = 1
period = 2π
The points remain the same as y=sin x.



max's occur at multiples of 2π.
min's occur at odd multiples of π.
zeros occur at any odd multiple of $\frac{\pi}{2}$.

Take a look at the graph of the cosine function y = cos x.

The amplitude, period, and maximum and minimum values of the cosine function are all the same as the corresponding values for the sine function.

Notice, though, that the locations of the maxima, minima, and zeros are different.