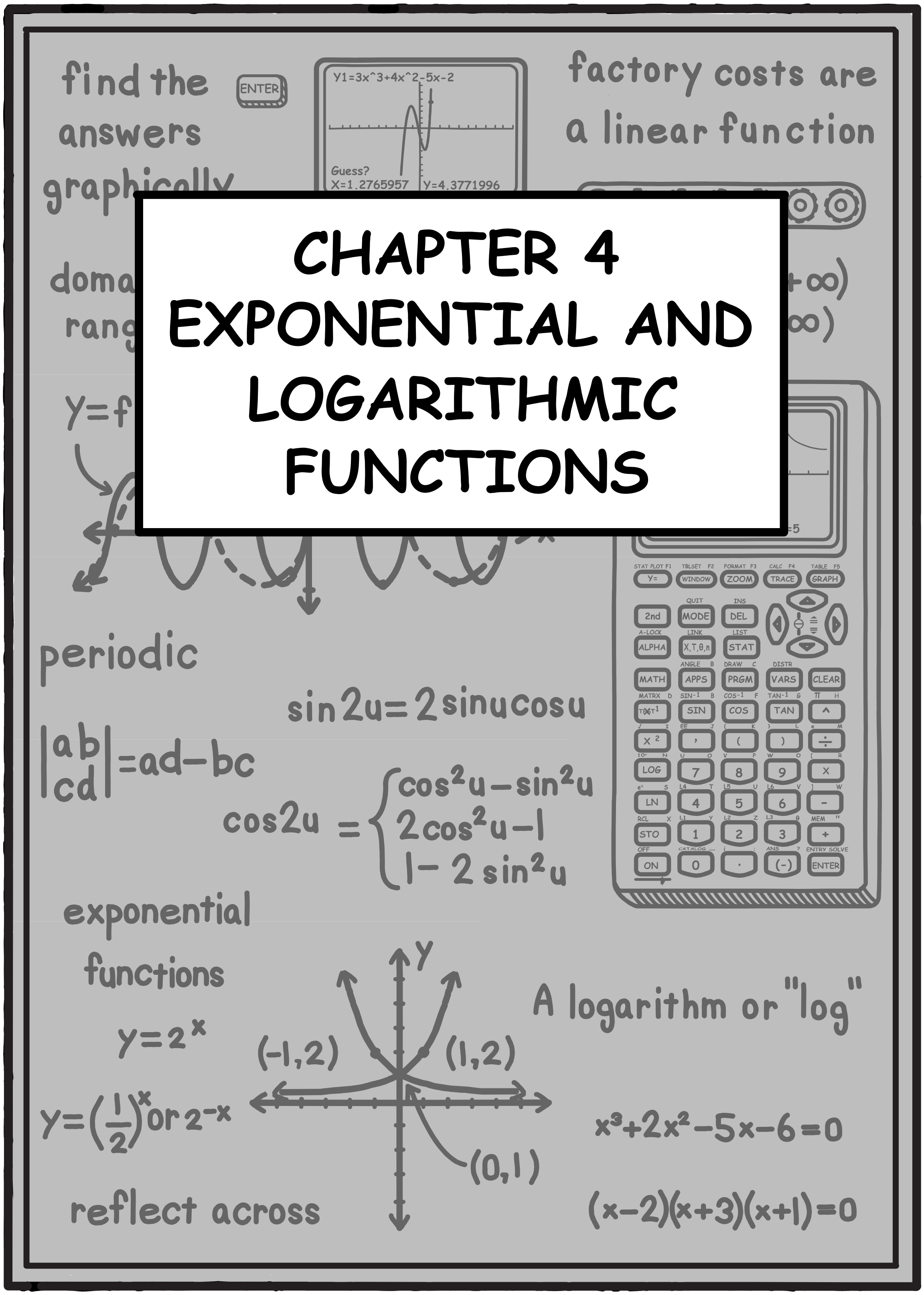
CHAPTER 4 EXPONENTIAL AND LOGARITHMIC FUNCTIONS



Lesson 23—Exponential Functions

So far we've learned about polynomial functions and rational functions. Another important category of functions are **exponential functions**. Those are functions where the variable is in the exponent.

Base Greater than 1

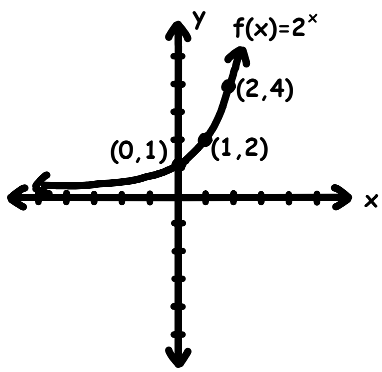
Here's a simple example: $f(x) = 2^x$. The independent variable x is in the exponent with a base of 2. That means $f(x)$ doubles every time x goes up by 1. The bigger $f(x)$ gets, the faster the function increases in value.

Table 23.1

$f(x)=2^x$	
$f(0)=2^0=1$	
	doubles from 1 to 2.
$f(1)=2^1=2$	
	doubles from 2 to 4
$f(2)=2^2=4$	
	doubles from 4 to 8
$f(3)=2^3=8$	
	doubles from 8 to 16
$f(4)=2^4=16$	

The graph of $f(x)$ can be seen below, along with a table of coordinates.

Figure 23.1



$f(x)=2^x$

x	y
3	8
2	4
1	2
0	1

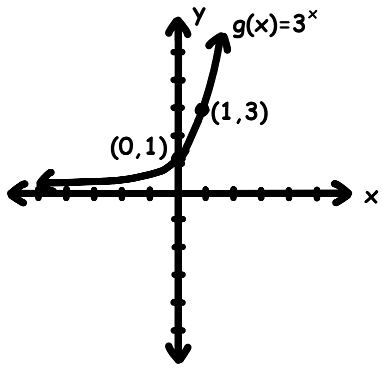
Notice that the graph curves upward, rising faster and faster as x increases. If we increase the base from 2 to 3, the function will increase even faster. Actually, it will triple every time x increases by 1.

Table 23.2

$g(x)=3^x$	
$g(0)=3^0=1$	
	triples from 1 to 3.
$g(1)=3^1=3$	
	triples from 3 to 9
$g(2)=3^2=9$	
	triples from 9 to 27
$g(3)=3^3=27$	
	triples from 27 to 81
$g(4)=3^4=81$	

Here's the graph of $g(x) = 3^x$, along with the graph of $f(x) = 2^x$. Notice that $g(x)$ rises even more steeply than $f(x)$.

Figure 23.2



g(x)=3^x

x	y
3	27
2	9
1	3
0	1

There can be all sorts of other exponential functions with different bases. The bigger the base, the faster the function will rise.

$h(x) = 4^x$ $\qquad$ $k(x) = 4.5^x$ $\qquad$ $j(x) = 5^x$ $\qquad$ $r(x) = 5.7^x$

Exponential functions like these all have certain things in common. First, their domain is equal to all real numbers. So any number can go in for *x*—positive or negative, rational or irrational.[1] Interestingly, when a negative number is put in for *x*, the function itself still comes out positive. For instance, $h(-2) = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$. That means the range of a function with the form $f(x) = a^x$ is all positive numbers.

Domain of f(x) = a^x = (-∞, +∞)

Range of f(x) = a^x = (0, +∞)

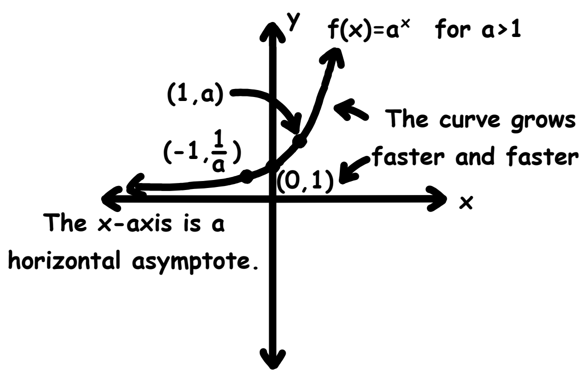
All exponential functions also include the point $(0,1)$, which is the *y*-intercept. That point has to be on every curve, because any power with an exponent of 0 automatically equals 1, regardless of the power's base. All exponential functions also include the point $x = 1$, $y = \text{the base}$. That's because the base raised to the first power equals the base itself. Using general symbols, we say that any function $f(x) = a^x$ goes through the point $(1, a)$, since $a^1 = a$. Another point that's included on the graph of any exponential function is $(-1, \frac{1}{a})$. Putting -1 in for $f(x) = a^x$, gives a^{-1}, which is the same as $\frac{1}{a}$.

Analyzing the end behavior of functions of the form $f(x) = a^x$ (where $a > 1$), we see that their value grows to positive infinity as *x* approaches positive infinity: $x \to +\infty$, $y \to +\infty$. As *x* approaches negative infinity, the value of each function gets closer and closer to 0: $x \to -\infty$, $y \to 0$ That means the *x*-axis ($y = 0$) is a horizontal asymptote. Since the curve never actually touches the *x*-axis, though, none of the exponential functions have any *x*-intercepts. None have any vertical asymptotes either.

[1] It was actually very challenging for the mathematicians to figure out that a variable in an exponent could equal an irrational number.

With all of this information, an exponential function of the form $f(x) = a^x$ is pretty easy to graph. The points $(0,1)$, $(1,a)$, and $(-1,\frac{1}{a})$ have to be on the curve. We know as well that the horizontal asymptote is at $y = 0$ and the curve goes upward toward the right at an increasing rate. That gives us the general shape of the graph.

Figure 23.3



Base Between 0 and 1

All of the examples so far have had a base that's greater than 1. But it's also possible for an exponential function to have a base that's between 0 and 1. Here are a couple of examples.

Table 23.3

$f(x)=(\frac{1}{2})^x$

x	y
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$

$g(x)=(\frac{1}{3})^x$

x	y
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$
3	$\frac{1}{27}$

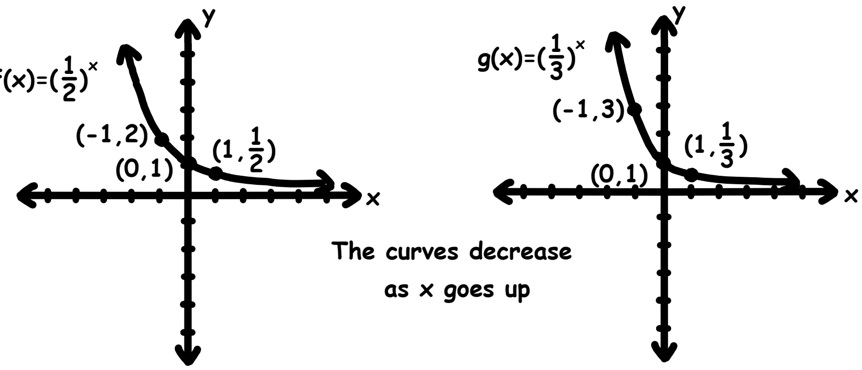
Notice that these functions go down as x increases. That's because instead of doubling or tripling each time, the function is multiplied by $\frac{1}{2}$ each time. Or in the case of $g(x) = \left(\frac{1}{3}\right)^x$, the function is multiplied by $\frac{1}{3}$ each time.

Table 23.4

$f(x)=(\frac{1}{2})^x$		$g(x)=(\frac{1}{3})^x$	
$f(0)=(\frac{1}{2})^0=1$		$g(0)=(\frac{1}{3})^0=1$	
	down to half from 1 to $\frac{1}{2}$.		down to third from 1 to $\frac{1}{3}$.
$f(1)=(\frac{1}{2})^1=\frac{1}{2}$		$g(1)=(\frac{1}{3})^1=\frac{1}{3}$	
	down to half from $\frac{1}{2}$ to $\frac{1}{4}$.		down to third from $\frac{1}{3}$ to $\frac{1}{9}$.
$f(2)=(\frac{1}{2})^2=\frac{1}{4}$		$g(2)=(\frac{1}{3})^2=\frac{1}{9}$	
	down to half from $\frac{1}{4}$ to $\frac{1}{8}$.		down to third from $\frac{1}{9}$ to $\frac{1}{27}$.
$f(3)=(\frac{1}{2})^3=\frac{1}{8}$		$g(3)=(\frac{1}{3})^3=\frac{1}{27}$	
	down to half from $\frac{1}{8}$ to $\frac{1}{16}$.		down to third from $\frac{1}{27}$ to $\frac{1}{81}$.
$f(4)=(\frac{1}{2})^4=\frac{1}{16}$		$g(4)=(\frac{1}{3})^4=\frac{1}{81}$	

Here are the graphs for these two functions. And the main thing to notice is that the graphs decrease as x goes up.

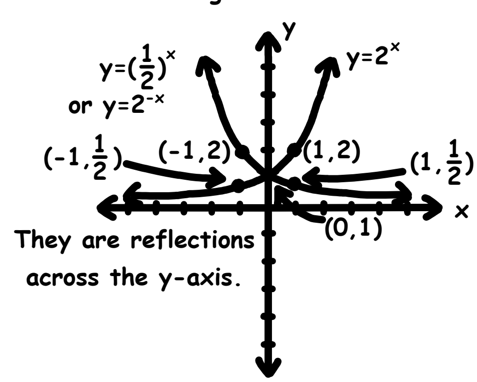
Figure 23.4



For any function $f(x)=a^x$ where $0<a<1$, the function will approach 0 as x approaches positive infinity. That means the x-axis ($y=0$) is still a horizontal asymptote. The domain is still all real numbers and the range is still all positive numbers, just like exponential functions where $a>1$. Each graph also still contains the points $(0,1)$, $(1,a)$, and $(-1,\frac{1}{a})$. Only since a is a fraction, $\frac{1}{a}$ turns out to be greater than 1. So there are many similarities between exponential functions with bases greater than 1 and those with bases between 0 and 1. By the way, we never have exponential functions with negative bases like $(-2)^x$. That would cause the function to have a lot of values that were not real numbers. For instance, when $x=\frac{1}{2}$, $(-2)^x$ turns out to equal an imaginary number: $(-2)^{\frac{1}{2}}=\sqrt{-2}=\sqrt{2}i$. We also don't allow an exponential function to have a base of 1. There's nothing wrong with $f(x)=1^x$. But since 1 raised to any power always equals 1, the function $f(x)=1^x$ is just the horizontal line $f(x)=1$, which isn't too helpful.

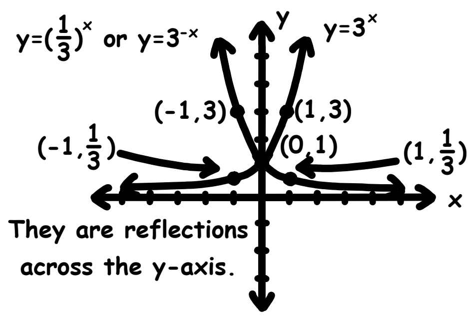
There's something else interesting about exponential functions that have a fraction for a base. Using the exponent rules from algebra, we can rewrite $\left(\frac{1}{2}\right)^x$ as $\frac{1^x}{2^x}$. But since 1 raised to any power equals 1, $\frac{1^x}{2^x}$ simplifies to $\frac{1}{2^x}$ or 2^{-x}. That means $\left(\frac{1}{2}\right)^x$ is the same as 2^{-x}. If you think back to transformations, changing a function $f(x)$ to $f(-x)$ results in reflecting the function across the y-axis. So 2^{-x} or $\left(\frac{1}{2}\right)^x$ is just 2^x reflected across the y-axis. Putting the graphs of both on the same coordinate plane shows this to be true (see right).

Figure 23.5



The same is true of $h(x) = 3^x$ and $g(x) = \left(\frac{1}{3}\right)^x$. Since $\left(\frac{1}{3}\right)^x$ can also be written as 3^{-x}, it is a reflection of 3^x across the y-axis.

Figure 23.6



Practice 23

a. Tell what happens to function $y = \left(\frac{1}{3}\right)^x$ as x increases by 1.

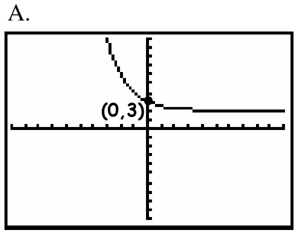
A. y triples B. y decreases by $\frac{1}{3}$ C. y stays constant

D. y increases by $\frac{1}{3}$ E. y is multiplied by $\frac{1}{3}$

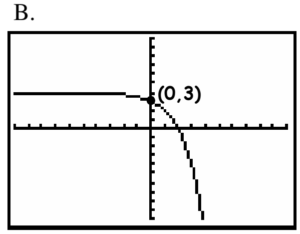
b. Calculate the value of the function $h(0) = 6^x$.

c. Select the graph for the exponential function $f(x) = 2^x - 3$.

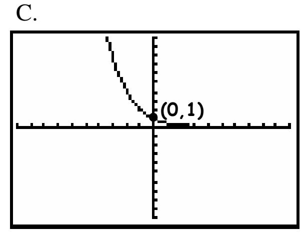
A.



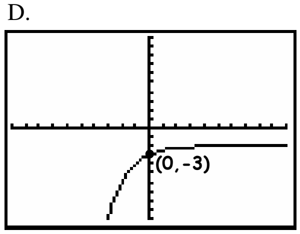
B.



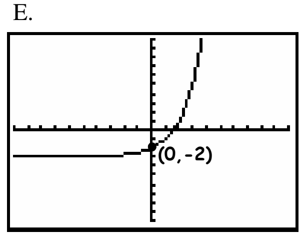
C.



D.



E.



Answer each question below.

d. What is the domain of the function: $f(x) = \dfrac{1}{2x^3 - 6x^2 - 8x}$?

A. $x < -1$ or $x > 4$ B. $\{x \mid -1 < x < 4\}$ C. $x \neq 0,\ 8$
D. $x \neq -1,\ 0,\ 4$ E. $x \geq 0$

e. Suppose the graph of $y = f(x)$ has a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = 4$. If $g(x) = \dfrac{1}{4} f(x)$, what are the vertical and horizontal aymptotes of the graph of $g(x)$?

Solve the word problem below.

f. As a memorial to the war hero, the city council of Eddington commissioned an architectural firm to design an arch to be placed in the center of the town square. The architects at the firm decided to build an arch whose shape can be represented by the function $f(x) = -\dfrac{1}{20}x^2 + 5x$ where x is in feet. Based on this function, how far apart are the bases of the arch?

Problem Set 23

Tell whether each sentence below is True or False.

1. An exponential function is a function like $f(x) = 5x^3$ that has an exponent.

2. When the base of an exponential function is greater than 1, the function increases as x approaches infinity.

Tell what happens to each function below as x increases by 1.

3. $y = 2^x$

A. y is cut in half B. y increases by 2 C. y decreases by 2
D. y stays constant E. y doubles

4. $y = 3^x$

A. y decreases by 3 B. y triples C. y stays constant
D. y is multiplied by $\dfrac{1}{3}$ E. y increases by 3

(a) 5. $y = \left(\frac{1}{2}\right)^x$

A. y is cut in half

B. y increases by $\frac{1}{2}$

C. y stays constant

D. y decreases by $\frac{1}{2}$

E. y doubles

Calculate the value of each function below.

6. $f(5) = 2^x$

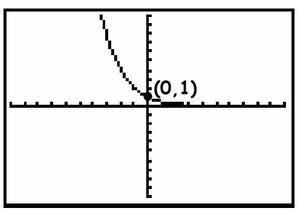
7. $g(2) = \left(\frac{1}{2}\right)^x$

(b) 8. $h(0) = 4^x$

Select the graph for each exponential function below.

9. $f(x) = 2^x$

A.



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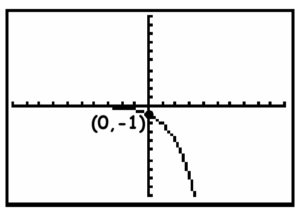
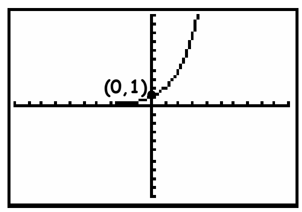
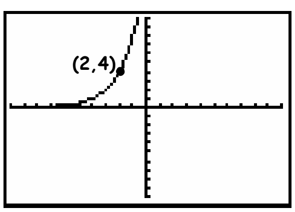


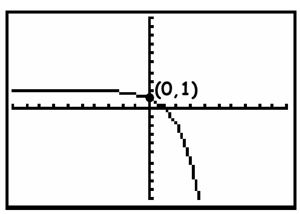
C.



D.

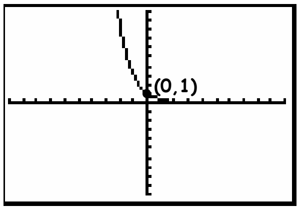


E.



10. $g(x)=\left(\dfrac{1}{3}\right)^{x}$

A.



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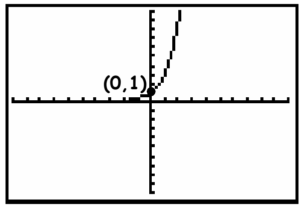
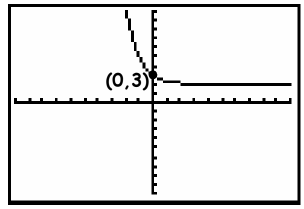
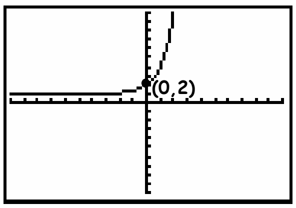


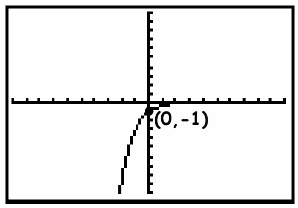
C.



D.

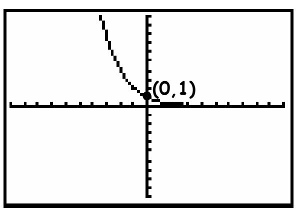


E.

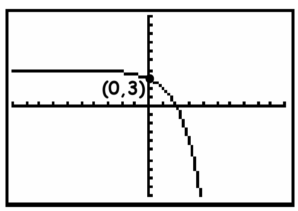


(c) 11. $f(x)=2^{x}-1$

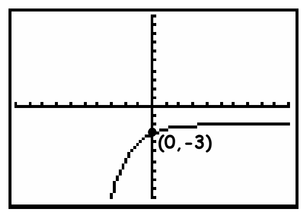
A.



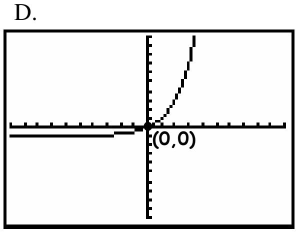
B.



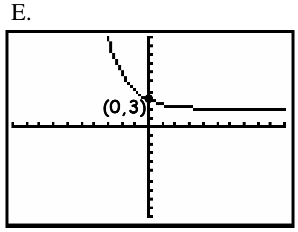
C.



D.



E.



Combine the complex numbers below.

12. $(1+4i)-(-2+2i)$

13. $(2-i)(1-3i)$

14. $\dfrac{-3+5i}{1-2i}$

Find the vertical asymptotes of each rational function below.

15. $f(x)=\dfrac{1}{2x^{2}-4}$

16. $g(x)=\dfrac{-x-2}{x^{2}-4x+3}$

Find the horizontal or slant asymptotes of each rational function below.

17. $m(x) = \dfrac{-2x^2 + 1}{x^2 - 3x}$

18. $n(x) = \dfrac{4x^2 - 7x + 6}{5x + 2}$

Answer each question below.

(d) 19. What is the domain of the function: $f(x) = \dfrac{1}{2x^3 + 2x^2 - 24x}$?

A. $x < -3$ or $x > 2$
B. $\{x \mid -3 < x < 4\}$
C. $x \neq -4, 0, 3$
D. $x \geq 0$
E. $x \neq 0, 4$

(e) 20. Suppose the graph of $y = f(x)$ has a vertical asymptote at $x = 6$ and a horizontal asymptote at $y = -1$. If $g(x) = \dfrac{1}{3} f(x)$, what are the vertical and horizontal asymptotes of the graph of $g(x)$?

Find the matching *x*-value for each function below using a graphing calculator. (Hint: Make sure the range of x and y-values is Xmin = -10, Xmax = 10, Ymin = -10, Ymax = 10.)

21. $y = 2x^3 - 3x^2 - 3x - 6$, $y = -8$, $x = ?$

22. $y = \dfrac{x^2 + 6x - 17}{x - 3}$, $y = 5$, $x = ?$

Solve the word problem below.

(f) 23. For his architectural class, Brad had to build a replica of the Gateway Arch, which is located in St. Louis. After finding the dimensions of the real arch, he found that his replica should follow the equation $f(x) = -\dfrac{4}{7}x^2 + 4x$ where *x* is in inches. Based on this function, how far apart are the bases of his arch?

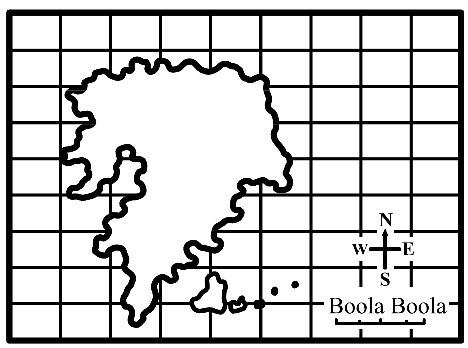
Lesson 24—Exponential Functions in the Real World

In the last lesson, we started learning about exponential functions. Remember, those are functions such as $f(x)=2^x$, where x is in the exponent. It turns out that exponential functions are very useful in the real world.

Population Growth

One common example involves population. The growth rate of a population usually depends on the population's size. The more people there are in a country, the more babies are going to be born. An exponential function represents this kind of growth extremely well. That's because in an exponential function, the growth rate is linked to the size of the function itself. When the function doubles, the rate of growth doubles, and when the function triples, the rate of growth triples, and so on. That's the kind of function needed to represent population growth, since a population that's twice as big should have roughly twice the growth rate (twice as many babies born). And a population that's three times as big should have about three times the growth rate.

Let's write an exponential function to represent the growth of the country Boola Boola.[2] Since we're tracking growth over time, the independent variable should be time (t) in years and the dependent variable should be the population's size (P). We'll track Boola Boola's growth starting from 2007, so that will be year 0 ($t=0$). In 2007 the population of Boola Boola was 12 million people.



It's tempting to write a function like $P=2^t$. But that won't work for a couple of reasons. Putting 0 for t in $P=2^t$ gives a population of 1: $1=2^0$, which is obviously wrong. The population of Boola Boola in 2007 was 12 million not 1. To fix this, we need to multiply 2^t by 12,000,000.

$$P=12{,}000{,}000(2^t)$$

Now putting 0 in for t makes P equal 12,000,000, which is what we want.

$$P=12{,}000{,}000(2^0)=12{,}000{,}000(1)=12{,}000{,}000$$

The function still isn't quite right, though. The problem is the base of 2. This base makes the function double every year. But Boola Boola couldn't possibly double in size every year. That would be incredibly fast growth for any country. If Boola Boola's population doubled every year, it would have the biggest population in the world in no time! We'll assume instead that the growth rate of Boola Boola is 3%. For 3% growth, we actually need a base of 1.03. The base should be 1 plus the growth rate written as a decimal. For a growth rate of 4%, we would change 4% to 0.04 and add it to 1 to get a base of 1.04. For a growth rate of 5%, we would add 0.05 to 1 to get a base of 1.05, and so on.

[2] In case you weren't sure, Boola Boola is a fictional country!

Here's why the base always has to be 1 plus the growth rate of the function. If Boola Boola is growing by 3% every year, then in 2008 its population can be calculated like this.

$$P \text{ in } 2008 = 12,000,000 + 12,000,000(0.03)$$

The right side starts with the original population of 12,000,000 and then adds 3% of that to get the new population for 2008. The expression can be simplified by factoring out (reverse distributing) 12,000,000.

$$P \text{ in } 2008 = 12,000,000(1+0.03)$$

Continuing the growth process, here's the calculation for the population in 2009.

$$P \text{ in } 2009 = 12,000,000(1+0.03) + 12,000,000(1+0.03)(0.03)$$

All we've done is start with the population in 2008, which is $12,000,000(1+0.03)$, and add 3% of that population to get the new population for 2009. The right side can be simplified again by factoring out $12,000,000(1+0.03)$.

$$P \text{ in } 2009 = 12,000,000(1+0.03)(1+0.03)$$

Since $(1+0.03)$ is multiplied twice at the end, we can write that as a square.

$$P \text{ in } 2009 = 12,000,000(1+0.03)^2$$

So the population of Boola Boola in 2008 can be written as $12,000,000(1+0.03)^1$ and the population in 2009 can be written as $12,000,000(1+0.03)^2$. After simplifying, the population for 2010 will have an exponent of 3 on $(1+0.03)$.

$$P \text{ in } 2010 = 12,000,000(1+0.03)^3$$

And the population for 2011, will have an exponent of 4.

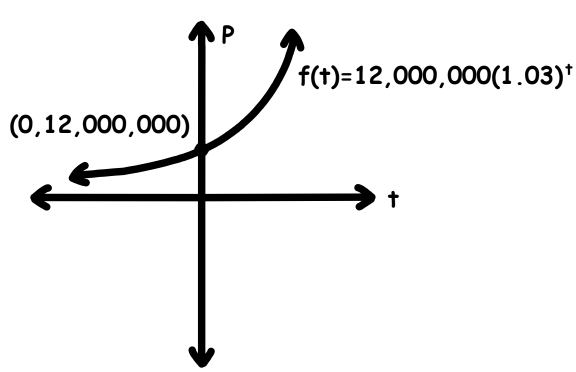
$$P \text{ in } 2010 = 12,000,000(1+0.03)^4$$

Are you starting to see the pattern? The exponent is always the year, which is t.

$$P = 12,000,000(1.03)^t$$

This function works exactly as it's supposed to. If the exponent t is 0, then $(1+0.03)^0$ equals 1 and the population will equal 12,000,000, which is the starting population. If $t=1$, then $(1+0.03)^1$ equals 1.03, and that gives a population 3% higher than 12,000,000, which is what it should be in the very next year. As t increases by 1 each year, another 3% is added to the population total. That's why the base needs to be 1 plus the growth rate. The graph of $P = 12,000,000(1.03)^t$ can be seen to the right. You can see that it goes up at an increasing rate to the right, just like all exponential functions (although we've had to adjust the P-axis, because the numbers are so big).

Figure 24.1



A More General Formula

We can use the same kind of function to represent different size populations that grow at different rates. The starting population is the population at $t = 0$. Putting 0 in the exponent always makes the function equal the constant (which is the number in front.) That constant must be the original population, then. We'll represent it with the letter P_0. (The little zero isn't an exponent. It just shows that this population number goes with $t = 0$.)

$$P = P_0(1.03)^t$$

To change the growth rate from 3% to some other rate, we just need to add the appropriate rate to 1. If we want a growth rate of 17%, then we should change 17% to 0.17 and add it to 1 to get 1.17. If we represent the decimal form of the growth rate with the letter *r* (for rate), then the base is always going to be $(1+r)$.

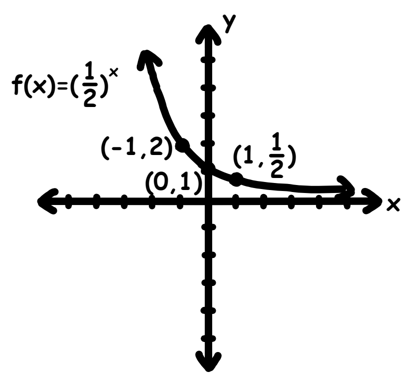
$$P = P_0(1+r)^t$$

This is the general form for an exponential function representing population growth. Actually, it can be used to represent anything that grows "exponentially." Technically, exponential growth happens only when the growth rate depends on the size of the function itself: when the function doubles, the growth rate doubles. That's when a function like $P = P_0(1+r)^t$ applies. In everyday life, people often say that something is "growing exponentially" when they really just mean that the quantity is growing fast. But fast growth isn't necessarily exponential growth.[3]

Exponential Decay

It's also possible for an exponential function to have a base between 0 and 1, remember. One simple example that we've already studied is $f(x) = \left(\frac{1}{2}\right)^x$. Any base that's between 0 and 1 makes the function go down as *x* increases.

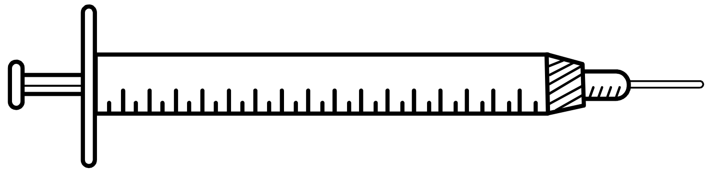
Figure 24.2



This kind of exponential function is sometimes referred to as "exponential decay," because the function gets smaller and smaller as *x* increases. It's the opposite of exponential growth. The rate of growth of an exponentially decaying function is actually negative. It's a rate of decline rather than a rate of growth. But that rate is still linked to the function itself. When the function's value is cut in half, the rate of decline will be cut in half.

[3] A function like $y = 95x^3$ grows fast, but it's not exponential. Eventually (when *x* gets large enough), an exponential function will grow faster than any monomial function. That's true no matter what the base of the exponential function (as long as it's greater than 1) and no matter how big the exponent on *x* in the monomial function.

There are a lot of real-world uses for exponential decay functions. For instance, when a doctor gives a patient a drug medication, there's quite a bit of the medication in the patient's bloodstream initially. But the amount of the medication declines over time. As it turns out, the decline can be represented by an exponential decay function. Here's a specific example.



> A doctor is giving a patient 5 milligrams of a particular drug and expects the amount of the drug in the patient's bloodstream to decrease by 33% each hour. Write a function to represent the relationship between the number of milligrams in the patient's bloodstream and the time since the drug was given.

We'll let M be the number of milligrams of the medication in the patient's bloodstream and t the number of hours after the drug was first given. That means instead of using the form $P = P_0(1+r)^t$, we should change it to $M = M_0(1+r)^t$. The letter M_0 is just the amount of the drug in the bloodstream at $t = 0$. That's 5 milligrams. The amount of the drug is supposed to decline by 33% each hour, so r should be a negative number. The growth rate is always negative with exponential decay functions. We need to change 33% to 0.33 and then make it negative: -0.33. Putting those values in the function gives us this.

$$M = 5(1+-0.33)^t$$

We can simplify by adding 1 and -0.33.

$$M = 5(0.67)^t$$

Notice that the base is now between 0 and 1. That's how we know it's an exponential decay function. Remember, exponential growth functions have a base that's greater than 1. Exponential decay involves a base that's between 0 and 1. The function $M = 5(0.67)^t$ can be used to figure out the amount of the drug in the patient's bloodstream after a certain number of hours. Let's find the amount after 3 hours. All we have to do is put 3 in for t and solve for M.

$$M = 5(0.67)^3$$

Since the base is a fraction, it will be easier if we use a calculator. We just punch in 0.67, then $\boxed{\wedge}$, then $\boxed{3}$, and then $\boxed{\text{ENTER}}$. That gives an answer of 0.300763. The last step is to multiply by 5, so we just press $\boxed{\times}$, then 5, and then $\boxed{\text{ENTER}}$ again. The answer comes out to 1.503815, which means that the amount of the drug in the patient's bloodstream will be down to about 1.5 milligrams after 3 hours.

Practice 24

Calculate the value of each function below.

a. $g(4) = \left(\dfrac{1}{3}\right)^{x-2}$

b. $m(3) = (2+1.5)^x$

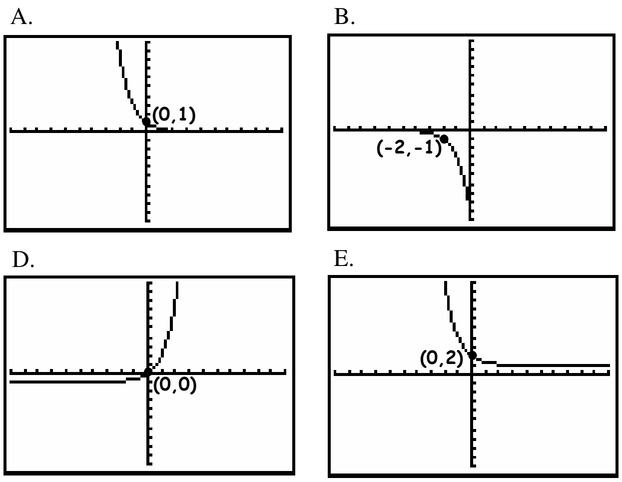
c. $n(2) = 4(2.75)^x$

d. The value of a laptop computer is \$3,780. A business owner estimates that the value of laptop computers depreciates at a rate of 27% per year. Select the function that represents the relationship between value of the laptop computer (P) and the number of years (t).

A. $P = 3,780(27)^t$ B. $P = (0.73)^t$ C. $P = 3,780(1.27)^t$

D. $P = (1.27)^t$ E. $P = 3,780(0.73)^t$

e. Select the graph for the exponential function $f(x) = 3^{x-2} + 1$.



Solve the word problem below.

f. When Cody went to the emergency room with a fractured femur (thigh bone), the doctors gave him a dose of long-lasting painkiller. If the doctors originally gave Cody 8 milligrams of the drug and expect the amount in his bloodstream to decrease by 15% each hour, how many milligrams of the drug will be in his bloodstream after 8 hours? Round your answer to two decimal places.

Problem Set 24

Tell whether each sentence below is True or False.

1. The function $P = P_0(1+r)^t$ can be used to represent the growth of a population where t is time in years, P_0 is the beginning population, and r is the growth rate (as a decimal).

2. An exponential decay function has a negative growth rate.

Tell what happens to each function below as x increases by 1.

3. $y = \left(\frac{1}{4}\right)^x$

A. y is multiplied by $\frac{1}{4}$ B. y increases by 4   C. y decreases by 4

D. y stays constant   E. y is multiplied by 4

4. $y = 5^x$

A. y decreases by 5 B. y is multiplied by 5 C. y stays constant

D. y is multiplied by $\frac{1}{5}$ E. y increases by 5

Calculate the value of each function below.

5. $f(-1) = 3^{x+1}$

(a) 6. $g(3) = \left(\frac{1}{2}\right)^{x-1}$

7. $h(1) = (1-0.25)^x$

(b) 8. $m(2) = (2+2.5)^x$

(c) 9. $n(2) = 3(1.75)^x$

Answer each question below.

10. Suppose that a bee colony starts out with 600 bees. A scientist estimates that the bee colony will increase at a rate of 16% per week. Select the function that represents the relationship between the size of the colony (P) and the number of weeks (t).

A. $P = (1.16)^t$ B. $P = 600(0.84)^t$ C. $P = 600(1.16)^t$

D. $P = (0.84)^t$ E. $P = 600(16)^t$

11. There are 120 bacteria in a Petri dish, and the bacteria are growing at a rate of 5% per hour. Select the function that represents the relationship between the number of bacteria in the Petri dish (P) and the number of hours (t).

A. $P = 120(5)^t$ B. $P = 120(0.95)^t$ C. $P = (0.95)^t$

D. $P = (1.5)^t$ E. $P = 120(1.05)^t$

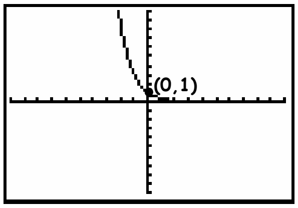
(d) 12. The atmospheric pressure at sea level is 100 kPa and the pressure decreases at a rate of 16% per kilometer above the sea level. Select the function that represents the relationship between the atmospheric pressure (P) and the number of kilometers (k).

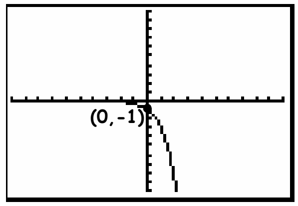
A. $P = 100(16)^k$ B. $P = 100(0.84)^k$ C. $P = (0.84)^k$

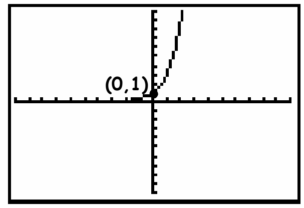
D. $P = (1.16)^k$ E. $P = 100(1.16)^k$

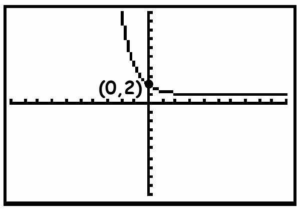
Select the graph for each exponential function below.

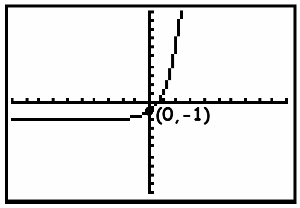
13. $f(x) = -3^x$

A.


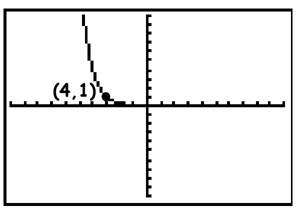
B.


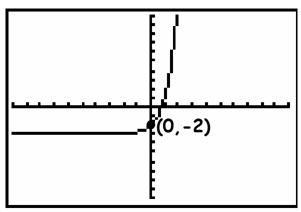
C.


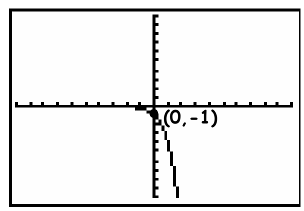
D.


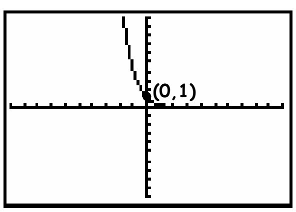
E.


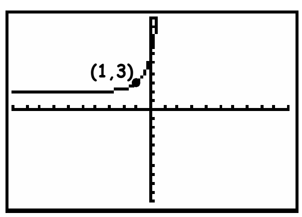
14. $g(x) = \left(\frac{1}{4}\right)^x$

A.


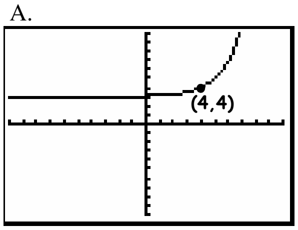
B.


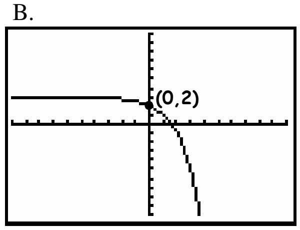
C.


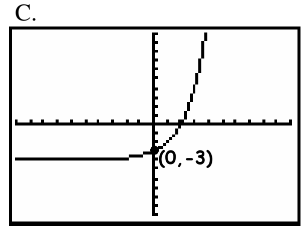
D.


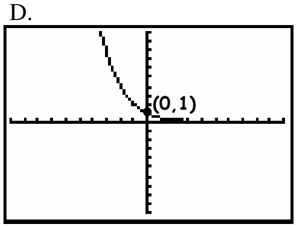
E.


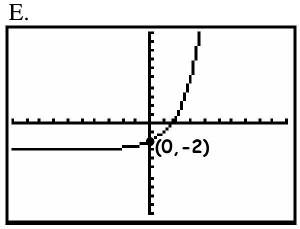
(e) 15. $f(x) = 2^{x-4} + 3$

A.


B.


C.


D.


E.


Factor each expression below.

16. $x^3 + 27$

17. $a^3 - 3a^2 + 3a - 1$

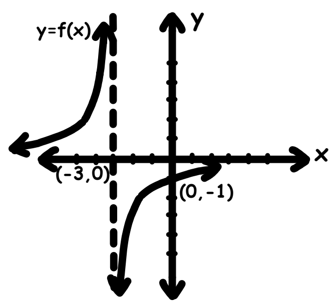
18. $a^3 - 8b^3$

Use a graphing calculator to graph each rational function below. Show the equation on the screen by pressing $\boxed{\text{TRACE}}$ after you're finished. (Hint: Adjust the range of x and y-values to Xmin = -10, Xmax = 10, Ymin = -15, Ymax = 15.)

19. $y = \dfrac{x^2+5}{x-2}$

20. $y = \dfrac{x^2-6x+9}{x^2-x-6}$

Select the rational function that represents each graph below.

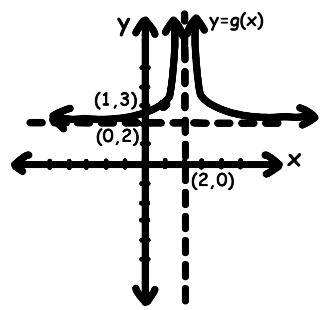


21. __________

A. $f(x) = -\dfrac{1}{x-2}$ B. $f(x) = \dfrac{-3}{x+3}$

C. $f(x) = \dfrac{2}{x+1}$ D. $f(x) = \dfrac{3}{x^2}$

E. $f(x) = \dfrac{-1}{(x+3)^2}$



22. __________

A. $g(x) = -\dfrac{1}{(x+2)^2}$ B. $g(x) = \dfrac{1}{x-2}$

C. $g(x) = \dfrac{-1}{x^2}$ D. $g(x) = \dfrac{2}{2x+1} + 2$

E. $g(x) = \dfrac{1}{(x-2)^2} + 2$

Answer the word problem below.

(f) 23. When Emily went to the emergency room, the doctors gave her a dose of blood coagulant, which is just a type of drug that helps blood clot. If the doctors originally gave Emily 4 milligrams of the drug and expect the amount in her bloodstream to decrease by 45% each hour, how many milligrams of the drug will be in her bloodstream after 6 hours? Round your answer to two decimal places.

Lesson 25—Logarithms

We've been learning about exponential functions, which are functions whose independent variable is in the exponent. The last problem set had some problems where you had to find a matching value for y given a value for x. One of the problems asked you to find $n(x)=3(1.75)^x$ when $x=2$.

Finding the Missing Exponent

But what about having a number for y and then finding the matching value for x? This is actually quite a bit tougher. To show you, let's find the matching value for x in $y=7(2)^x$ when $y=112$. Putting 112 in for y gives us this.

$$112=7(2)^x$$

Now we have an equation where the unknown is in the exponent. That's what makes it tough. As a first step, we could undo the multiplication of 7 by dividing both sides by 7.

$$\frac{112}{7}=\frac{7(2)^x}{7}$$

$$16=2^x$$

But how do we get the x out of the exponent? As it turns out, adding, subtracting, multiplying, or dividing both sides of the equation won't work. Neither will taking a root or raising both sides to a power. We can still solve the equation, though. What we can do is rewrite 16 as a power with a base of 2. The power 2^2 is too small because it equals 4. And 2^3 is also too small, because it equals 8. But 2^4 equals 16 exactly, so it works. Therefore, we need to change 16 to 2^4 in the equation.

$$2^4=2^x$$

Now think about this. The only way the powers 2^4 and 2^x can be equal is if their exponents are equal. So we can write another equation with just the exponents.

$$4=x$$

That gives us an answer of 4. If you put 4 back in for x in the original equation, you'll see that it works.

Equations with an x in the exponent like $112=7(2)^x$ are called exponential equations. They're really just exponential functions with a number put in for y. One way to solve an exponential equation is to rewrite it so that both sides are a power with the same base. Then you can create another, simpler equation by setting the exponents equal to each other. Let's do another example.

$$4\left(\frac{1}{2}\right)^{-3x}=\frac{1}{8}$$

It's tempting to multiply the 4 and $\frac{1}{2}$, but that's wrong. The order of operations rules says that powers are supposed to be done before multiplication. So $\frac{1}{2}$ can't be split away from its exponent. However, we can undo the multiplication of 4 by dividing both sides by 4.

$$\left(\frac{1}{2}\right)^{-3x}=\frac{1}{32}$$

Now we need to rewrite $\frac{1}{32}$ as a power with a base of $\frac{1}{2}$. Since $32 = 2^5$, the fraction $\frac{1}{32}$ is the same as $\frac{1}{2^5}$. But $\frac{1}{2^5}$ can be changed to $\frac{1^5}{2^5}$ or $\left(\frac{1}{2}\right)^5$. Changing $\frac{1}{32}$ to $\left(\frac{1}{2}\right)^5$ gives us this.

$$\left(\frac{1}{2}\right)^{-3x} = \left(\frac{1}{2}\right)^5$$

Now, since the powers are equal and their bases are the same, the exponents also must be equal.

$$-3x = 5$$

The last step is to solve this simple equation by undoing.

$$x = -\frac{5}{3}$$

You can check this answer, if you want, by substituting it back into the original equation.

A Tough Exponent to Find

Unfortunately, not all exponential equations can be solved by coming up with exponents in your head. Look at this equation.

$$91 = 7(10)^x$$

We can divide both sides by 7 first.

$$13 = 10^x$$

The next step is to rewrite 13 as a power with a base of 10. But what should the exponent be? It can't be a whole number, because 10^1 equals 10, which is too low. And 10^2 equals 100, which is too high. It must be some exponent between 1 and 2. One thing is for sure. You won't be able to come up with the exponent in your head.

The way to find this missing exponent is to use something called a logarithm. A logarithm, or "log" for short, is really just a fancy name for an exponent that goes on a base to get a certain answer. For instance, the logarithm of 100 is the exponent that goes on 10 to get 100. Since $10^2 = 100$, the log of 100 equals 2. Similarly, the log of 1,000 is the exponent that goes on 10 to get 1,000. Since $10^3 = 1{,}000$, the log of 100 equals 3. Some logs can be done in your head like this, but most can't. In the old days, to find logs you had to use a table of logarithms, which was just a page filled with numbers showing lots of different exponents. Today logs can be found really fast with a calculator.

To finish solving our equation $13 = 10^x$, we need to find the log of 13. It's actually written as $\log 13$. That's the exponent that goes on 10 to get 13. To find $\log 13$ on a calculator, we first punch the $\boxed{\text{LOG}}$ key. (On the TI-83 Plus, it's on the far left a little more than halfway down.) The calculator will show "log" followed by an open parenthesis. It looks like this.

Next, we punch in the number 13 followed by a close parenthesis: $\boxed{)}$. The last step is to press $\boxed{\text{ENTER}}$. The answer comes out to 1.113943352. That's the exponent that goes on 10 to get 13. In other words, $10^{1.113943352}$ must equal 13. Actually, even this long decimal is just an estimate. The real exponent is an irrational number with digits that go on forever. We'll round down even further to 4 decimal places to get $10^{1.1139}$. That gives us the following equation.

$$10^{1.1139} = 10^x$$

The next step is pretty easy. Since the powers now have the same base, their exponents must be equal.

$$1.1139 = x$$

The approximate answer to the equation is 1.1139. It's approximate because we rounded. But that's how to use logs to solve a tough exponential equation.

Other Bases

It's also possible to have logs of other bases besides 10. The expression $\log_2 16$ is asking for the exponent that goes on a base of 2 to get 16. Since $2^4 = 16$, the answer is 4. So $\log_2 16$ is a base 2 log. Notice the little 2 below. That's how you know. Here are some examples of logs with different bases.

$\log_4 64 = 3$ (since $4^3 = 64$) $\log_3 81 = 4$ (since $3^4 = 81$)

$\log_7 7 = 1$ (since $7^1 = 7$) $\log_5 1 = 0$ (since $5^0 = 1$)

We could write base 10 logs as $\log_{10}$. The little 10 is usually left off, though, because base 10 logs are used so often. A base 10 log, then, is just written as "log." That's why in the last example we wrote the base 10 log of 13 as just $\log 13$. It's kind of like writing a square root as $\sqrt{4}$ instead of $\sqrt[2]{4}$. Actually, since base 10 logs are so "common," they're also called **common logarithms**. So $\log 13$ is called "the common log of 13." One important thing to remember is that the $\boxed{\text{LOG}}$ key on your calculator is for only base 10.

Taking the Log of Both Sides

There's a more formal way to solve exponential equations using logs. Take the equation below as an example.

$$10^x = 8$$

To find the answer, we need the common log of 8 ($\log 8$), which is the exponent you put on 10 to get 8. Instead of figuring that out with the calculator and substituting the power for 8 on the right side, most people take the common log of both sides.

$$\log 10^x = \log 8$$

On the right side, we have $\log 8$, which is what we need to figure out. The left side is asking for the exponent that you put on 10 to get 10^x. But that's just x. So the left side can be simplified to x.

$$x = \log 8$$

To get the answer, we just punch in $\log 8$ on the calculator. Rounded to 4 decimal places, that comes out to this.

$$x = 0.9031$$

We would have gotten the same thing by just replacing 8 with $10^{0.9031}$ in $10^x = 8$ to get $10^x = 10^{0.9031}$, and then setting the exponents equal to each other. But taking the log of both sides is kind of like undoing, and it makes solving harder exponential equations easier. Just remember that taking the log of both sides can get an x out of an exponent. It "undoes" or gets rid of the base 10.

Practice 25

a. Find the logarithm $\log_{10} 10{,}000$ without using a calculator.

b. Use a calculator to find the logarithm $\log_{10} 7$. Estimate the answer to four decimal places.

Find the matching value in each exponential function below. Round any irrational answers to four decimal places.

c. $2(3)^x = 54$, $x = ?$

d. $9(10)^x = 126$, $x = ?$

e. Select the domain and range of the exponential function $y = 5^x + 1$.

A. Domain: $x \geq 0$; Range: $y \geq 1$

B. Domain: $\{x \mid x \leq 0\}$; Range: $(-\infty,\ 1]$

C. Domain: $(-\infty,\ +\infty)$; Range: $(-\infty,\ +\infty)$

D. Domain: All real numbers; Range: $y \geq 0$

E. Domain: $(-\infty,\ +\infty)$; Range: $\{y \mid y > 1\}$

f. Solve the word problem below.

Donna just bought a new truck for \$18,950. If the truck decreases in value at a rate of 8% per year, how much will it be worth in 4 years? Round your answer to the nearest cent.

Problem Set 25

Tell whether each sentence below is True or False.

1. Equations with an x in the exponent are called exponential equations.

2. A logarithm is a fancy name for an exponent that goes on a base to get a certain answer.

Find the horizontal or slant asymptotes of each rational function below.

3. $f(x) = \dfrac{x^2 - 4}{-x^3 + 5x}$

4. $g(x) = \dfrac{-x^2 + 8x - 4}{3x + 6}$

Find each logarithm below without using a calculator.

5. $\log_2 16$

6. $\log_3 27$

(a) 7. $\log 100$

Use a calculator to find each logarithm below. Estimate each answer to four decimal places.

8. $\log 12$

9. $\log 150$

(b) 10. $\log 3$

Find the matching value in each exponential function below. Round any irrational answers to four decimal places.

11. $y = \left(\frac{1}{3}\right)^{x-2}$, $x = 4,\ y = ?$

(c) 12. $9(2)^x = 36$, $x = ?$

13. $6(10)^x = 48$, $x = ?$

(d) 14. $7(10)^x = 98$, $x = ?$

Answer each question below.

15. Select the inverse function of $h(x) = -4x + 3$.

A. $-\frac{x+3}{4}$

B. $\frac{3-x}{4}$

C. $\frac{x-4}{3}$

D. $\frac{-x-3}{-4}$

E. $\frac{3x}{4}$

16. If $f(x) = 2x$ and $g(x) = \frac{x^2}{5}$, what is $f^{-1}\left(g^{-1}(5)\right)$?

Select the domain and range of each exponential function below.

17. $y = \left(\frac{1}{2}\right)^x$

A. Domain: $\{x \mid x \geq 0\}$; Range: $y \geq 0$

B. Domain: $(-\infty,\ 0]$; Range: $[0,\ +\infty)$

C. Domain: $(-\infty,\ +\infty)$; Range: $(-\infty,\ +\infty)$

D. Domain: $(-\infty,\ +\infty)$; Range: $\{y \mid y > 0\}$

E. Domain: $x \leq 0$; Range: $y \leq 0$

(e) 18. $y = -2^x$

A. Domain: $(-\infty,\ +\infty)$; Range: $\{y \mid y \ge 0\}$

B. Domain: $[0,\ +\infty)$; Range: $(-\infty,\ 0]$

C. Domain: $-\infty < x < +\infty$; Range: $-\infty < y < +\infty$

D. Domain: $\{x \mid x \ge 0\}$; Range: $y \le 0$

E. Domain: $(-\infty,\ +\infty)$; Range: $(-\infty,\ 0)$

Answer each question below.

19. The starting salary for a firefighter in Pleasant Hill is 43,000 dollars. Firefighters get an increase in their salary of 6% each year. Select the function that represents the relationship between a Pleasant Hill firefighter's salary (S) and the number of years on the job (t).

A. $S = 43{,}000(6)^t$

B. $S = (0.94)^t$

C. $P = 43{,}000(0.94)^t$

D. $S = 43{,}000(1.06)^t$

E. $S = (1.6)^t$

20. If the initial temperature of in an electric oven is $95\ °\mathrm{C}$ and its temperature decreases 27% every minute after it is turned off. Select the function that represents the relationship between the temperature in the oven in $°\mathrm{C}$ (T) and the number of minutes (t).

A. $T = 95(1.27)^t$

B. $T = 95(0.73)^t$

C. $T = 95(27)^t$

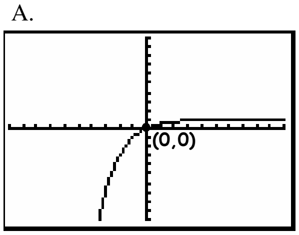
D. $T = (0.73)^t$

E. $T = (1.27)^t$

Select the graph for each exponential function below.

21. $f(x) = \left(\dfrac{1}{2}\right)^{-x}$

A.



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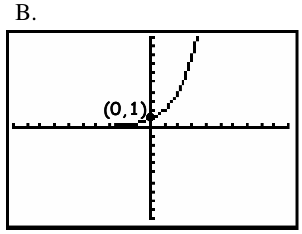

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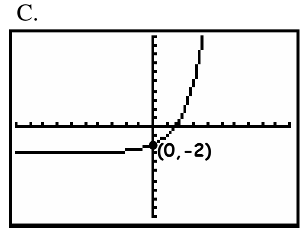

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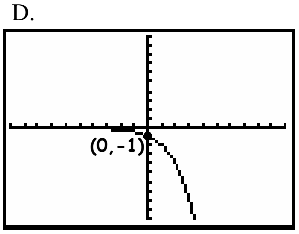

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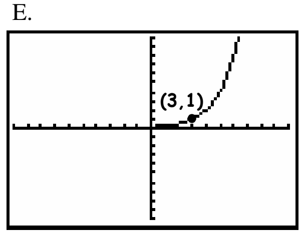
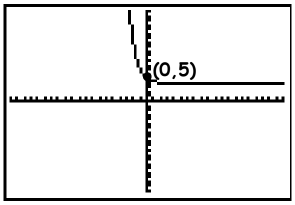
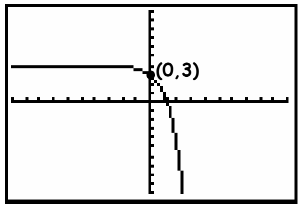


22. $g(x) = 3^{x-1} + 4$

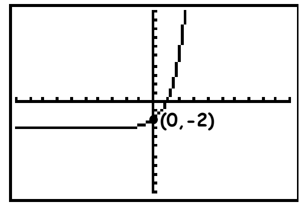
A.



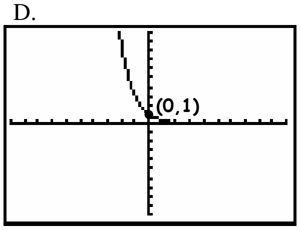
B.



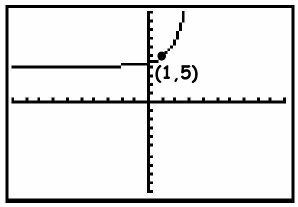
C.



D.



E.



Solve the word problem below.

(f) 23. Stephen just bought a brand new sports car for \$43,500. If the car decreases in value at a rate of 9% per year, how much will it be worth in 6 years? Round your answer to the nearest cent.

Lesson 26—More on Logarithms

In the last lesson, we started learning about logarithms. Remember, a logarithm is just an exponent that goes on a base to get a certain answer. Logs can be used to solve exponential equations, where x is in the exponent.

Other Bases Besides 10

Some exponential equations are more difficult, because the power with x in its exponent has a base other than 10. Here's an example.

$$5^{2x-1} = 3$$

If our calculator had a base 5 log key, we could find the exponent that goes on 5 to get 3 ($\log_5 3$). With that exponent, we could make both of the bases the same.

$$5^{2x-1} = 5^{?}$$

We need the log of 3 for base 5.

But since calculators don't have base 5 log keys, what we have to do is change both bases to 10. The first step is to find the common log of 5. We just press $\boxed{\text{LOG}}$, then $\boxed{5}$, then $\boxed{)}$, then $\boxed{\text{ENTER}}$. The answer comes out to 0.6989700043 or 0.6990 rounded to 4 decimal places. That means $10^{0.6990} = 5$ (approximately). Next, we find the common log of 3. We press $\boxed{\text{LOG}}$, then $\boxed{3}$, then $\boxed{)}$, then $\boxed{\text{ENTER}}$ to get 0.4771 (rounded). That tells us $10^{0.4771} = 3$ (approximately). Now we can substitute these powers for 5 and for 3 in the equation.

$$(10^{0.6990})^{2x-1} = 10^{0.4771}$$

If you remember the rules for handling exponents from algebra, when a power is raised to another power, the exponents can be multiplied. A simple example is $(2^3)^4 = 2^{3 \cdot 4} = 2^{12}$. We can apply this rule to the left side of our equation by multiplying 0.6990 and $(2x-1)$.

$$10^{0.6990(2x-1)} = 10^{0.4771}$$

Since the powers have the same base, we can set the exponents equal to each other.

$$0.6990(2x-1) = 0.4771$$

From here, we just solve the equation in the usual way (and round to 4 decimal places).

$$2x - 1 = 0.6825$$

$$2x = 1.6825$$

$$x = 0.8413$$

You can check this answer by substituting 0.8413 back in for x in the original equation. It should make the sides close to equal, although it won't be exact because of rounding.

We also could have solved $5^{2x-1} = 3$ in the more formal way by taking the log of both sides.

$$\log 5^{2x-1} = \log 3$$

We already figured out that $\log 3 = 0.4771$. So that gives us this.

$$\log 5^{2x-1} = 0.4771$$

But what about the left side? This is asking for the exponent that you put on 10 (since log is base 10) to get 5^{2x-1}. There's an important property of logarithms which says that any expression of the form $\log a^b$ can be rewritten as the exponent times the log of the base. This is sometimes called the power rule for logs.[4]

$$\log a^b = b \log a \qquad\qquad (a > 0)$$

By using this rule, we can rewrite the left side of $\log 5^{2x-1} = 0.4771$ like this.

$$(2x-1)\log 5 = 0.4771$$

To find $\log 5$, we just punch that into a calculator. The answer comes out to be 0.6990 (rounded).

$$(2x-1)(0.6990) = 0.4771$$

This is the same equation we got before, and so the answer has to be $x = 0.8413$ again.

Some Basic Logs

There are some logs that are used so often that it's worth memorizing them. For instance, what if we take the log of a number *b* in the same base *b*? That gives $\log_b b$. This is asking what exponent goes on *b* to get *b*. The answer has to be 1, since $b^1 = b$. That means $\log_b b = 1$. This works no matter what the particular base of the log. It could be $\log_2 2$, $\log_7 7$, $\log_{18} 18$ or anything else. The answer is always 1.

Another basic log is the log of 1. As it turns out, no matter what the base, the log of 1 always equals 0. So we have $\log_b 1 = 0$. This also works for any base, because *any* number raised to the zero power automatically equals 1.

Here's a third basic log that comes up a lot. Let's take the log in base 2 of the power 2^5. That gives $\log_2 2^5$, which is asking for the exponent that goes on 2 to get 2^5. The answer is obviously 5. So $\log_2 2^5 = 5$. Notice the answer is just the exponent on 2^5. It works the same way for any base and any exponent. If you take the log of any power in the same base, the answer will always equal the exponent. We can write it like this: $\log_b b^a = a$. Think of the log as undoing the base and leaving just the exponent.

---

[4] Here's why this rule works. The expression $\log a^b$ is asking for the exponent that you put on 10 to get a^b. We'll call that exponent *k*. So $k = \log a^b$ and $10^k = a^b$. The log of *a* is some other exponent that if put it on a base of 10 will equal *a*. We'll call that exponent *p*. That means $10^k = (10^p)^b$. Multiplying exponents, we get $10^k = 10^{bp}$. But the exponents have to be equal, so $k = bp$. Since $p = \log a$, we have $k = b\log a$. But $k = \log a^b$, so $\log a^b = b\log a$.

Our fourth basic log shows that the undoing process also works in the other direction. Take the expression $3^{\log_3 7}$, for example, The base of the power is 3. The exponent is a number that when placed as an exponent on 3 equals 7. Think about what that means. If the exponent $\log_3 7$ makes a base of 3 equal 7, then $3^{\log_3 7}$ must also equal 7. So for the expression $3^{\log_3 7}$, the log undoes the power, leaving just 7. Keep in mind, though, that this only works when the bases are the same. It has to be 3 raised to a log of 3 or 4 raised to a log of 4, etc. The general rule is written like this: $b^{\log_b a} = a$.

Here is a table of all these basic logs (plus the power rule from the last lesson). One side shows the basic logs for any base and the other side shows them for base 10 (which is how you'll probably use them most often). Notice also that the power rule applies to any base, not just to base 10.

Table 26.1
Basic Log Rules

1.	$\log_b b=1$	$\log 10=1$
2.	$\log_b 1=0$	$\log 1=0$
3.	$\log_b b^y=y$	$\log 10^y=y$
4.	$b^{\log_b x}=x$	$10^{\log x}=x$
5.	$\log_b M^r=r\log_b M$	$\log M^r=r\log M$

For 0<b≠1, M>0, x>0, and any real number r

Change of Base Formula

Since calculators don't have log keys for every base, we need a fast way to find a log no matter what its base happens to be. Otherwise, how would we calculate $\log_4 9$? There's no log key for base 4 on a calculator. There's actually a way to figure out a log of any base using just the common log (base 10) key. All we have to do is find the common log of 9 and the common log of 4. Since both those logs are base 10, we can use the $\boxed{\text{LOG}}$ key. Then we just divide those like this.

$$\log_4 9 = \frac{\log 9}{\log 4}$$

$$\log_4 9 = \frac{0.9542}{0.6021}$$

$$\log_4 9 = 1.5848$$

The answer comes out to be 1.5848, so that's the exponent that goes on 4 to equal 9. The neat thing about this shortcut is that it will work for any base. The shortcut is called the change of base formula, because it allows you to find a log with a different base than base 10. Here's the formula stated generally.

$$\log_b M = \frac{\log M}{\log b}$$ **change of base formula**

It's pretty easy to show where the change of base formula comes from. What if we didn't know the formula but still needed to figure out $\log_4 9$? This is asking for the exponent that goes on 4 to get 9, remember. That can be written as an equation where x is the missing exponent.

$$4^x = 9$$

Now we can solve this equation by taking the common log (base 10) of both sides.

$$\log 4^x = \log 9$$

Using the power rule ($\log a^b = b \log a$), we can bring the x down from the exponent and multiply it by log4.

$$x \log 4 = \log 9$$

The last step is to divide both sides by log4 by get x by itself.

$$x = \frac{\log 9}{\log 4}$$

This is the change of base formula. That's where it comes from. It's just solving for the missing exponent by taking the common log of both sides of an exponential equation. Only instead of having to solve an exponential equation every time, you can just use the formula. It's a lot faster. Just remember that in the formula the log of the base always goes on bottom.

Practice 26

a. Solve the exponential equation $\left(\frac{1}{4}\right)^{-x+3} = 64$ by setting the exponents equal to each other.

b. Find the matching value in the exponential function: $9\left(\frac{1}{3}\right)^{-2x} = \frac{1}{27}$, $x = ?$

c. Use the change of base formula to find $\log_7 35$. Estimate your answer to four decimal places.

Solve each exponential equation below by taking the log of both sides. Estimate your answers to four decimal places.

d. $7^{5x+2} = 3$

e. $\left(\frac{1}{6}\right)^{3-7x} = 4$

Solve the word problem below.

f. Tinyville has a modest population of 50,000, but it has an annual growth rate of 18%. At that rate, how many years will it take to for the population to double in size? Round your answer to two decimal places.

Problem Set 26

Tell whether each sentence below is True or False.

1. To solve an exponential equation, take the log of both sides.

2. You can find $\log_3 15$ by calculating $\dfrac{\log 3}{\log 15}$.

Find each logarithm below without using a calculator.

3. $\log_2 2^5$

4. $3^{\log_3 8}$

5. $10^{\log 7}$

Use a calculator to find each logarithm below. Estimate each answer to four decimal places.

6. $\log 4$

7. $\log 9$

Solve each exponential equation below by setting the exponents equal to each other.

8. $7^{2x+1} = 49$

(a) 9. $\left(\dfrac{1}{5}\right)^{-x+2} = 25$

10. $3^{1-2x} = \dfrac{1}{3}$

Find the matching value in each exponential function below. Round any irrational answers to four decimal places.

11. $y = 3^{-x+1},\ x = 2,\ y = ?$

(b) 12. $8\left(\dfrac{1}{2}\right)^{-2x} = \dfrac{1}{4},\ x = ?$

13. $7(10)^x = 105,\ x = ?$

Use the change of base formula to find each log below. Estimate your answer to four decimal places.

14. $\log_2 5$

15. $\log_3 10$

(c) 16. $\log_6 24$

Solve each exponential equation below by taking the log of both sides. Round any irrational number to four decimal places.

17. $3^x = 7$

(d) 18. $6^{3x+1} = 4$

(e) 19. $\left(\frac{1}{5}\right)^{4-2x} = 2$

Select the domain and range of each exponential function below.

20. $y = 4\left(\frac{1}{5}\right)^{x}$

A. Domain: $\{x \mid x \geq 0\}$; Range: $y \geq 4$

B. Domain: $-\infty < y < +\infty$; Range: $\{y \mid y \geq 0\}$

C. Domain: $(-\infty, +\infty)$; Range: $(0, +\infty)$

D. Domain: $(-\infty, +\infty)$; Range: $(-\infty, +\infty)$

E. Domain: $[0, +\infty)$; Range: $[4, +\infty)$

21. $y = 2^{x-3} - 1$

A. Domain: $(-\infty, +\infty)$; Range: $\{y \mid y > -1\}$

B. Domain: $[0, +\infty)$; Range: $(-\infty, -1]$

C. Domain: All real numbers; Range: All real numbers

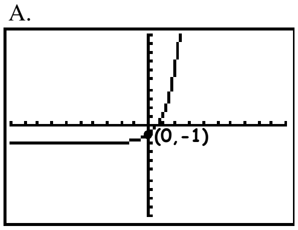
D. Domain: $\{x \mid x \geq 0\}$; Range: $y \geq -1$

E. Domain: $(-\infty, +\infty)$; Range: $[-1, +\infty)$

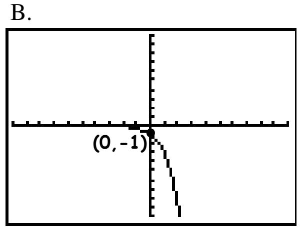
Select the graph for each exponential function below.

22. $f(x) = 3^{-x} - 2$

A.



B.



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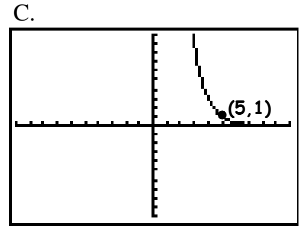
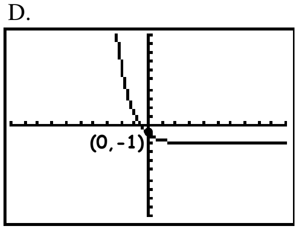


D.



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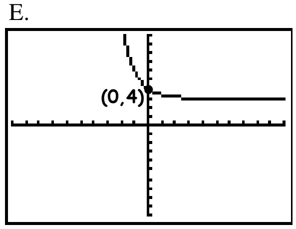
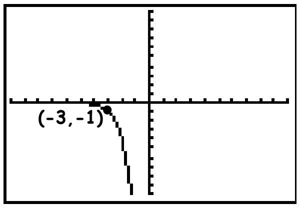
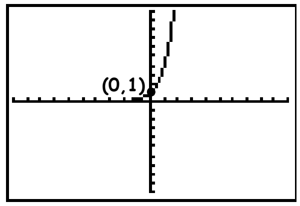


23. $g(x) = -4^{x+3}$

A.



B.



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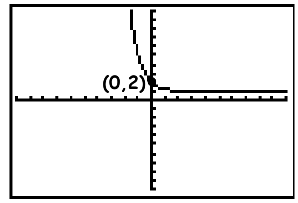
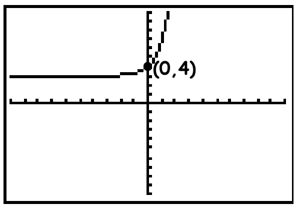
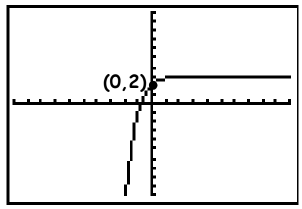


D.



E.



Solve the word problem below.

(f) 24. Harrington is a large city with a population of about 350,000. A recently taken census found that the annual growth rate of the city is 6%. If it continues to grow at this rate how many years will it take for the population to triple in size? Round your answer to two decimal places.

Lesson 27—Logarithmic Functions

We've been learning about logs. They can be used to solve exponential equations, where x is in the exponent. As it turns out, logs can also be used in other ways. Maybe most importantly, they can be turned into functions.

Inverse of the Exponential Function

We know how to find the inverse of a function. For instance, to find the inverse of $f(x) = 2x + 5$, we just solve for x in $y = 2x + 5$ and then switch the positions of x and y.

$$y = 2x + 5$$

$$\frac{y-5}{2} = x$$

$$\frac{x-5}{2} = y$$

$$f^{-1}(x) = \frac{x-5}{2}$$

We can find the inverse of an exponential function in the same way. Let's find the inverse of $g(x) = 10^x$. To solve for x, we can just take the log of both sides.

$$\log y = \log 10^x$$

The right side is asking for the log of the quantity 10^x, which is the exponent that goes on 10 to get 10^x. This is one of the basic logs we learned about in the last lesson. Remember, since the base of the log and the base of the power are both 10, the log undoes the power, leaving just x.

$$\log y = x$$

Now that the equation is solved for x, the last step in finding the inverse is to switch x and y.

$$\log x = y$$

$$y = \log x$$

The inverse of the exponential function $g(x) = 10^x$ is a new kind of function $g^{-1}(x) = \log x$. Since this function takes the log of x, it's called a logarithmic function or log function for short.[5] The function $g^{-1}(x) = \log x$ can be used in the same ways we have used lots of other functions in this course. For instance, we can put a number like 3 in for x and find its matching value. All we have to do is punch $\log 3$ into a calculator.

$$g^{-1}(3) = \log 3 = 0.4771$$

For $g^{-1}(x)$, when $x = 3$, $y = 0.4771$.

[5] Since exponential functions are one-to-one, the inverse is itself a function. In other words, it has just one y-value for each x-value.

A log function also has a domain and range. The domain of $y = \log x$ is all positive real numbers and the range is all real numbers (both positive and negative). That makes sense, because the domain and range are switched for the inverse of a function (because the *x*'s and *y*'s are switched). Remember, the domain of $y = 10^x$ is all real numbers and the range is all positive real numbers. So for $y = \log x$, which is the inverse, the domain is all positive real numbers and the range is all real numbers.

$$\text{Domain of } y = \log x \text{ equals } (0, +\infty)$$

$$\text{Range of } y = \log x \text{ equals } (-\infty, +\infty)$$

Log Form vs. Exponential Form

We can also solve for the other variable (*x*) in a log function. For example, let's find *x* when $y = 4$ in the function $y = \log x$. First, we put 4 in for *y*.

$$4 = \log x$$

This equation says that 4 is the exponent that goes on 10 (since this is a base 10 log) to get some number *x*. We can write the same thing using an exponent.

$$4 = \log x \text{ can also be written as } 10^4 = x$$

Since $10^4 = 10{,}000$, *x* must equal 10,000.

We say that $4 = \log x$ is the log form of the equation and $10^4 = x$ is the exponential form. Actually, going from $4 = \log x$ to $10^4 = x$ is just turning each side of $4 = \log x$ into an exponent. We can make 4 and $\log x$ exponents, each on a base of 10.

$$10^4 = 10^{\log x}$$

Now the right side is one of the basic logs we've already learned about. The base and log undo each other leaving just *x*.

$$10^4 = x$$

Most people just go directly from $4 = \log x$ to $10^4 = x$ without the extra step of putting in the bases. The main point is that we can find a matching value for *x* given a value for *y* in a log function by changing it to exponential form.

It's also possible to go in the other direction. For instance, we can start with an equation like $10^y = 5$ and change it to log form. The equation says that when the exponent *y* goes on 10, the result equals 5. That's the same thing as saying that the log (base 10) of 5 equals *y*. So $10^y = 5$ can also be written as $y = \log 5$.

Different Bases

Log functions can have other bases besides 10 as well. The function $y = \log_2 x$ is a log function in base 2. This means that *y* is the exponent that must be placed on a base of 2 to get *x*. So the exponential form of $y = \log_2 x$ is $2^y = x$. Here are several other log functions with different bases.

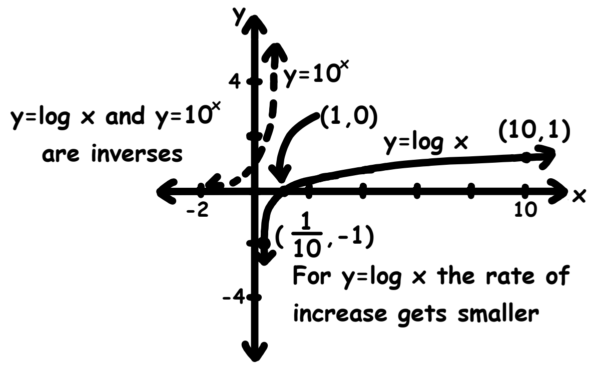
$y = \log_5 x$ which means $5^y = x$ $\qquad$ $y = \log_8 x$ which means $8^y = x$

$y = \log_7 x$ which means $7^y = x$ $\qquad$ $y = \log_{13} x$ which means $13^y = x$

Graphs of Log Functions

What about the graph of a log function? Well, $y = \log x$ is the inverse of the exponential function $y = 10^x$. And, remember, the inverse is a reflection of the original function across the line $y = x$. So the graph of $y = \log x$ has to be a mirror image of $y = 10^x$ across $y = x$.

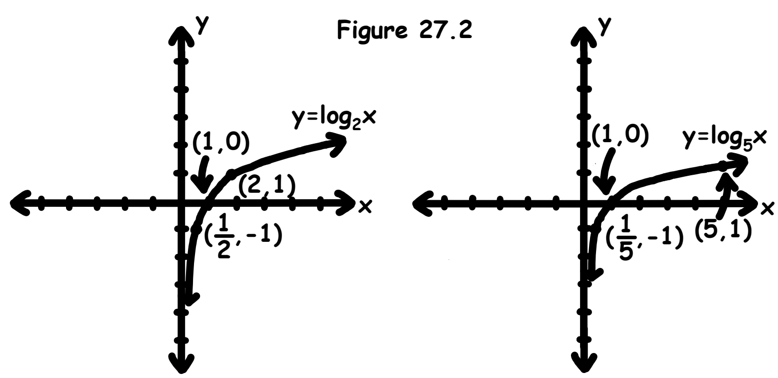
Figure 27.1



Notice that $y = \log x$ rises as x increases. But look at the way the curve bends away from $y = 10^x$. The curve of $y = 10^x$ bends upward because its rate of increase gets bigger and bigger as x approaches infinity. But the curve of $y = \log x$ bends on the horizontal direction. That's because its rate of increase actually gets smaller and smaller as x approaches infinity. That's the big difference between exponential and log functions. If a person says that a certain quantity has "logarithmic growth," he means that the quantity increases over time, but at a slower and slower rate.

The graph of $y = \log x$ intersects the point $(1,0)$ and $(10,1)$, and $(\frac{1}{10}, -1)$. As it turns out, the graph of any log function of the form $y = \log_a x$ will always intersect $(1,0)$, $(a,1)$, and $(\frac{1}{a}, -1)$, where a is the base. Here are graphs of two log functions with specific bases of 2 and 5.

Figure 27.2



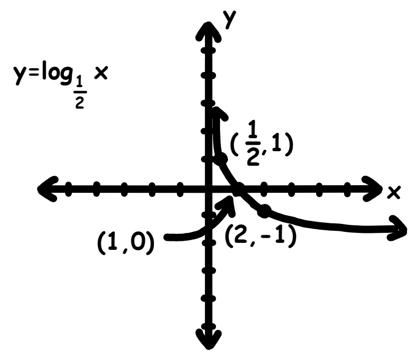
Notice that the log base 2 graph crosses the points $(1,0)$, $(2,1)$, and $(\frac{1}{2}, -1)$ and the log base 5 graph crosses the points $(1,0)$, $(5,1)$, and $(\frac{1}{5}, -1)$. That's the pattern.

It's interesting to compare the three points $(1,0)$, $(a,1)$, and $(\frac{1}{a},-1)$, which we use to graph $y=\log_a x$, to the three points we use to graph an exponential function of the form $y=a^x$. For $y=a^x$, we use the $(0,1)$, $(1,a)$, and $(-1,\frac{1}{a})$. The x and y have been switched. That's the only difference between the points for $y=a^x$ and $y=\log_a x$, which makes perfect sense because $y=a^x$ and $y=\log_a x$ are inverses.

Fractional Bases

Some log functions have a fractional base like $\frac{1}{2}$ or $\frac{1}{3}$. The function $y=\log_{\frac{1}{2}} x$ means that y is the exponent that must be placed on a base of $\frac{1}{2}$ to get x. Here's the equation in exponential form: $\left(\frac{1}{2}\right)^y = x$. The function $y=\log_{\frac{1}{3}} x$ works the same way. It's exponential form is $\left(\frac{1}{3}\right)^y = x$. Log functions with fractional bases aren't used as much as the regular log functions. But the most interesting thing about them is their graphs. The graph of $y=\log_a x$ where $0<a<1$ actually goes down as x increases. The key points are still $(1,0)$, $(a,1)$, and $(\frac{1}{a},-1)$. But since a is itself a fraction, $\frac{1}{a}$ ends up being greater than 1. Here's the graph of $y=\log_{\frac{1}{2}} x$.

Figure 27.3



Practice 27

a. For which of the following is the range of $f(x)$ equal to the set of all positive numbers?

I. $f(x)=x^2+3$ II. $f(x)=5^{x+1}$ III. $f(x)=1-3^x$

A. I only B. II only C. II and III
D. I and II E. I, II, and III

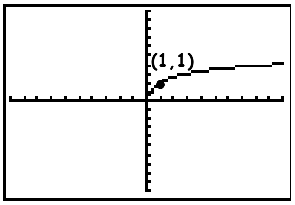
b. Rewrite the exponential equation $x^5=15$ into log form.

c. Rewrite the log equation $\log_4(x-3)=7$ into exponential form.

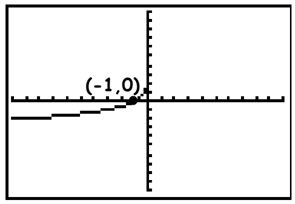
d. Find $7^{\log_7 4}$.

e. Select the graph for the log function: $y=\log_{\frac{1}{3}} x$.

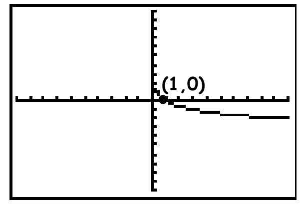
A.



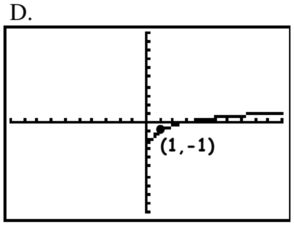
B.



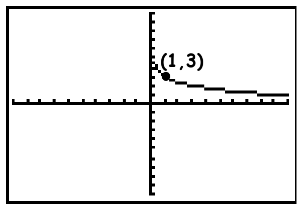
C.



D.



E.



f. Solve the word problem below.

Silvertown grew rapidly to a population of 20,000 during the recent gold rush, but now that all of the gold has been mined, the town is shrinking at a rate of 4% annually. At that rate, how many people will be left in the town after 5 years? Round your answer to the nearest whole number.

Problem Set 27

Tell whether each sentence below is True or False.

1. The inverse of the exponential function $y=10^x$ is the log function $y=\log x$.

2. The graph of $y=\log x$ rises faster and faster as x approaches infinity.

Solve each exponential equation below by setting the exponents equal to each other.

3. $6^{3x+4}=36$

4. $\left(\frac{1}{3}\right)^{-x+7}=81$

Answer each question below.

5. $\log_x x = ?$

(a) 6. For which of the following is the range of $f(x)$ equal to the set of all positive numbers?

I. $f(x) = x^2 + 2$ II. $f(x) = 2^{x+2}$ III. $f(x) = 1 - 2^x$

A. I only B. II only C. I and II
D. II and III E. I, II, and III

Solve each exponential equation below by taking the log of both sides. Estimate your answer to four decimal places.

7. $5^x = 9$

8. $7^{6x+3} = 2$

9. $\left(\frac{1}{8}\right)^{1-4x} = 6$

Rewrite each exponential equation into log form.

10. $6^x = 15$

11. $3^5 = x$

(b) 12. $x^4 = 20$

Rewrite each log equation into exponential form.

13. $\log x = 7$

14. $\log_2 9 = x$

(c) 15. $\log_3(x+2) = 5$

Find each log below.

16. $\log_4 4^3$

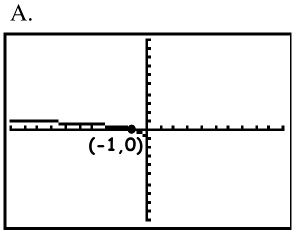
17. $\log 1$

(d) 18. $5^{\log_5 6}$

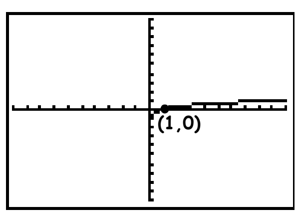
Select the graph for each log function below.

19. $y = \log x$

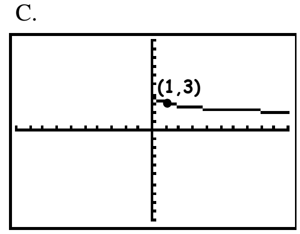
A.



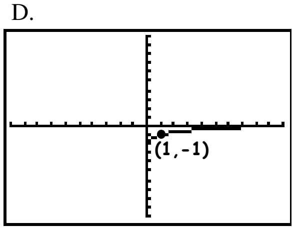
B.



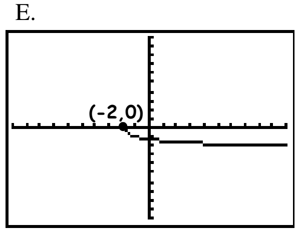
C.



D.

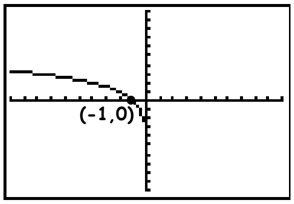


E.

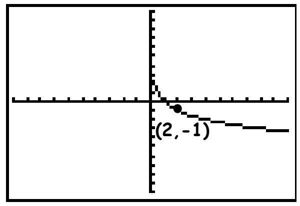


20. $y = \log_2 x$

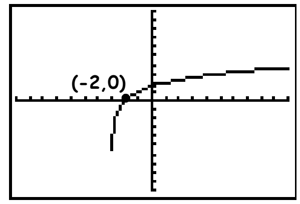
A.



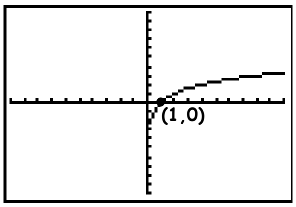
B.



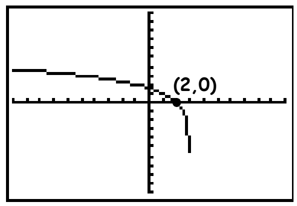
C.



D.

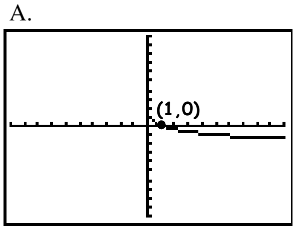


E.



21. $y = -\log_5 x$

A.



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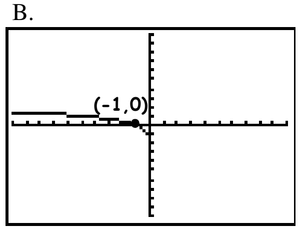

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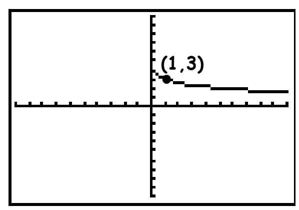
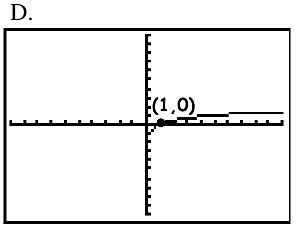
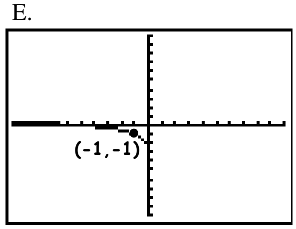


D.

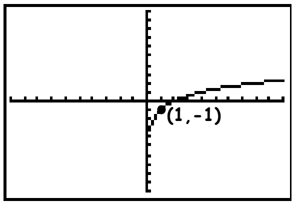


E.

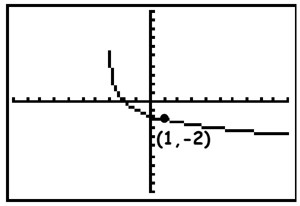


(e) 22. $y = \log_{\frac{1}{2}} x$

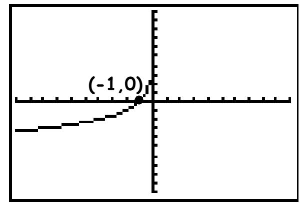
A.



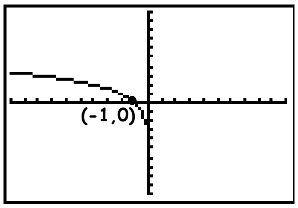
B.



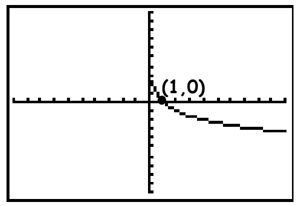
C.



D.



E.



Solve the word problem below.

(f) 23. The recent closure of a large manufacturing plant in Grandburgh has caused many families to relocate. If the city originally had a population of 50,000 and it is shrinking at a rate of 5% annually, how many people will still live in Grandburgh after 10 years? Round your answer to the nearest whole number.

Lesson 28—Solving Log Equations

When working with log functions, it's pretty common to put a number in for y and then find the matching value for x. For example, in the function $y = \log x$, we might need to find x when $y = 2$. To do that, we first put 2 in for y.

$$2 = \log x$$

Then we can solve for x by rewriting the equation in exponential form.

$$10^2 = x$$

Since 10^2 equals 100, when $y = 2$, $x = 100$.

Harder Log Equations

The equation $2 = \log x$ is called a log equation, because the unknown x is contained in the log. Since $2 = \log x$ is simple, it can be solved by just rewriting the equation into exponential form. But some harder log equations can also be solved that way. Take this example.

$$\log(2x - 4) = 3$$

In this equation, the log is not of x but of the quantity $(2x - 4)$. To rewrite this in exponential form, we just need to realize that the equation is saying that when 3 becomes an exponent on 10, the result equals $(2x - 4)$. Written with an exponent, it looks like this.

$$10^3 = 2x - 4$$

Now we can solve normally.

$$1{,}000 = 2x - 4$$

$$1{,}004 = 2x$$

$$502 = x$$

The answer is 502. On harder log equations, it's very important to check your answer. The reason is that it's impossible to take a log of a negative number (since the log function is only defined for positive real numbers). So we have to make sure that there's no negative log when putting the answer back in for x in the original equation. Putting 502 in $\log(2x - 4) = 3$ gives us this.

$$\log[2(502) - 4] = 3$$ **checking our answer**

Simplifying inside the brackets, we get

$$\log(1{,}000) = 3$$

$$3 = 3$$

The log is positive and the left side comes out to 3, so we know the answer 502 is correct. By the way, the number that you're taking the log of in an expression is called the argument. So the argument of $\log x$ is x and the argument of $\log(2x - 4)$ is $2x - 4$. We always have to check the answer to a log equation to make sure that the argument is positive. Let's do another example.

$$\log_3(x - 7) = \log_3 9$$

At first, this equation seems tougher, because the logs are base 3 instead of base 10. But since both logs have the same base, we can just set the arguments equal to each other.

$$x-7=9$$

Solving this little equation gives $x=16$. We should check this answer, though. Putting 16 back into the original equation we get this.

$$\log_3(16-7)=\log_3 9$$

$$\log_3 9=\log_3 9$$

The argument is positive and the answer checks out. The main point of this last example is that when both logs have the same base, you can just set the arguments equal to each other. It's kind of like setting the exponents equal to each other in an exponential equation. Remember, we can solve an equation like $2^{-4x}=2^{x+6}$ by setting the exponents equal to get $-4x=x+6$.

The Power Rule Backwards

Some log equations are too hard to solve by just putting them into exponential form or setting the arguments equal to each other. Here's a tougher one.

$$2\log x=\log 16$$

Both of these logs are common logs (base 10). But we can't just set the arguments equal to each other, because $\log x$ is multiplied by 2. Instead of dividing both sides by 2 and calculating $\frac{\log 16}{2}$, it's faster to use the power rule in reverse. Remember, the power rule (for base 10) says that $\log a^b=b\log a$. In $2\log x$, the 2 is in the b position and the x is in the a position. Applying the power rule backwards, $2\log x$ is the same as $\log x^2$. That allows us to rewrite our equation like this.

$$\log x^2=\log 16$$

Now we have just two common logs that are equal to each other. And we can set the arguments equal to each other.

$$x^2=16$$

Taking the square root of both sides, we end up with two answers.

$$x=+4,\ -4$$

We have to check these answers in the original equation to make sure that both make the argument positive.

$$2\log x=\log 16$$

checking +4	**checking -4**
$2\log 4=\log 16$	$2\log(-4)=\log 16$
$1.2041=1.2041$	**negative argument**

The answer +4 checks out, but -4 makes the argument negative, so -4 is an extraneous solution that has to be thrown out. The only valid answer to the equation, then, is 4.

The Product Rule

Here's a log equation that requires using another important rule.

$$\log_4(x+3)+\log_4(2-x)=1$$

These are both logs of base 4, but they're added to each other. Moving one to the other side of the equation wouldn't do any good because of the 1. We still wouldn't be able to set the arguments equal to each other. We need to use a log rule called the product rule on this one. As it turns out, two logs added together are the same as the log of their arguments multiplied.

$$\log_b MN = \log_b M + \log_b N \quad (M > 0,\ N > 0,\ b > 0,\ b \neq 1)$$

This looks complicated, but it actually makes sense. According to the exponent rules from algebra, when two powers of the same base are multiplied you can just add their exponents. For instance, $10^2 \cdot 10^3$ equals 10^{2+3} or 10^5. Well, the product rule is saying basically the same thing. It says that the exponent of the power M ($\log M$) added to the exponent of the power N ($\log N$) is equal to the exponent of the power that equals M times N. Using numbers, The 2 from 10^2 added to the 3 from 10^3 must equal the 5 from 10^5.

$$\log 10^2 \cdot 10^3 = \log 10^2 + \log 10^3$$

Getting back to our equation, $\log_4(x+3)+\log_4(2-x)=1$, since the two logs that are added have the same base, we can use the product rule to rewrite the left side like this.

$$\log_4(x+3)(2-x)=1$$

Now there's just one log. And that allows us to rewrite the equation in exponential form.

$$4^1=(x+3)(2-x)$$

Next, we simplify on both sides.

$$4=-x^2-x+6$$

$$x^2+x-2=0$$

We now have a quadratic equation that can be solved by factoring.

$$(x+2)(x-1)=0$$

$$x=-2\ ,\ 1$$

That gives us two answers: -2 and 1. We won't go through the checking process, but both answers cause the arguments to be positive. So -2 and 1 are the final solutions.

The Quotient Rule

There's another log rule called the quotient rule, which also relies on the algebra rules for exponents. Remember, you can divide two powers of the same base by subtracting their exponents. For instance, $\frac{10^7}{10^3}$ equals 10^{7-4} or 10^3. That means two logs subtracted are the same as the log of their arguments divided.

$$\log_b \frac{M}{N} = \log_b M - \log_b N \quad (M > 0,\ N > 0,\ b > 0,\ b \neq 1)$$

The quotient rule can be used on equations such as this.

$$\log_3(x-1) - \log_3(x+6) = \log_3(x-2) - \log_3(x+3)$$

This looks tough, but notice that every log is base 3. That means if we could write both sides as a single log, then we could set the arguments equal to each other. Since the logs are subtracted on both sides, we can use the quotient rule.

$$\log_3 \frac{x-1}{x+6} = \log_3 \frac{x-2}{x+3}$$

The arguments are just the fractions in each log. Setting those equals gives us this.

$$\frac{x-1}{x+6} = \frac{x-2}{x+3}$$

Now we have a rational equation that can be solved in the usual way. Instead of clearing the fractions by multiplying both sides by $(x+6)(x+3)$, it's a little faster to cross multiply.

$$(x-1)(x+3) = (x+6)(x-2)$$

$$x^2 + 2x - 3 = x^2 + 4x - 12$$

Subtracting x^2 from both sides and simplifying gives us an answer of $\frac{9}{2}$.

$$9 = 2x$$

$$x = \frac{9}{2}$$

Of course, $\frac{9}{2}$ has to be checked by putting it back into the original equation. We won't go through the steps, but the answer does turn out to be valid.

Solving Log Equations by Graphing

One of the toughest things about solving log equations is checking your answers. You never know when an extraneous solution might pop up. Even worse, if you make certain kinds of mistakes while solving, it's possible to lose some valid solutions. Then you wouldn't end up finding every solution to the equation. One way to avoid such errors is to solve a log equation with a graphing calculator.[6] To show how it works, let's solve $5\log x^2 = 3$ with that approach.

One way to solve $5\log x^2 = 3$ by graphing is to change $5\log x^2 = 3$ to $5\log x^2 - 3 = 0$. Then you can graph $y = 5\log x^2 - 3$ and find the zeros of that function. That's the same method we use to find tough polynomial equations by graphing. Another approach, though, is to think of $5\log x^2 = 3$ as two separate functions set equal to each other. So $f(x) = 5\log x^2$ is a function, and $g(x) = 3$ is another function. We can then graph both $f(x)$ and $g(x)$ on the same coordinate plane, using the calculator. The intersection points are the solutions to the equation. Those points lie on both graphs, so that's where both functions have the same value. Either approach will work. You can graph $y = 5\log x^2 - 3$ and find its zeros or graph $f(x) = 5\log x^2$ and $g(x) = 3$ separately. Let's go through the steps for the second approach, so you'll understand how it's done. The first step is to graph $f(x) = 5\log x^2$. Here are the keys that need to be punched.

Enter the function 5log x²

1) press Y=

2) press 5

3) press LOG

4) press X,T,θ,n

5) press ^

6) press 2

7) press)

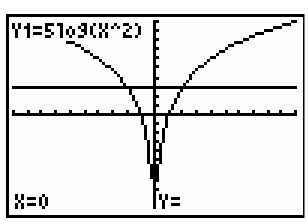
Next, we press the down arrow key. That takes us to $y_2 =$, which is the place for entering in a second function. To enter in $g(x)$, we just press 3.

Go to y_2 = and Enter the function 3

1) press ▼

2) press 3

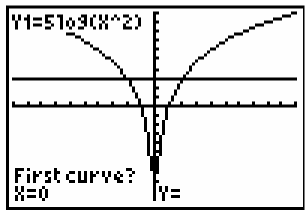
Next, we press $\boxed{\text{GRAPH}}$, then $\boxed{\text{TRACE}}$. That shows both graphs on the same coordinate plane. The graph of $5\log x^2$ is symmetrical about the *y*-axis and, of course, the graph of $y = 3$ is a horizontal line. They intersect at two points, one on either side of the *y*-axis. That shows that the equation $5\log x^2 = 3$ must have two solutions.



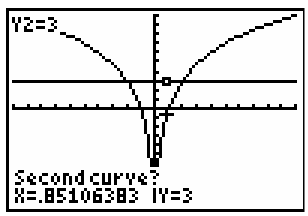
[6] Solving with a graphing calculator is the only method for really tough log equations that can't be solved at all with algebra techniques.

Notice that only one of the equations is showing in the upper left. To get the other equation to show, we need to press the down arrow.

Now we have to find the intersection points. To do that, we press $\boxed{\text{2nd}}$, then $\boxed{\text{TRACE}}$. That's actually the CALC (calculate) key, which is in yellow above $\boxed{\text{TRACE}}$. The calculate menu then comes up. We select choice 5, which is "intersect." That causes the calculator to ask "First curve?"

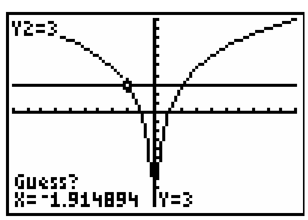


Since $y_1 = 5\log(x^2)$ is in the upper left, we'll let that be the first curve. The way to do it is to move the left or right arrow keys until we see that the cursor is on the curve $5\log(x^2)$. Then we press $\boxed{\text{ENTER}}$. That causes the calculator to ask "Second curve?" The equation $y_2 = 3$ should be in the upper left.

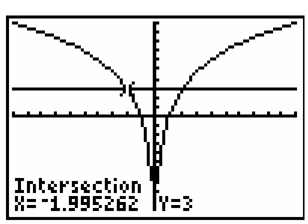


Next, we press the left or right arrow keys to make sure that the cursor is on the line $y_2 = 3$, and then we press $\boxed{\text{ENTER}}$ again. Now the calculator asks us to guess one of the intersection points.

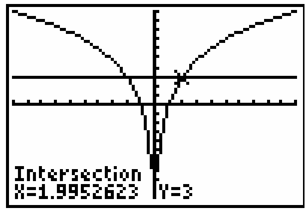
We'll guess the intersection point on the left by moving the cursor (using the left or right arrow keys) until it's as close to that point as possible and then pressing $\boxed{\text{ENTER}}$.



That gives us the calculator's estimate of the first intersection point at $x = -1.995262$ and $y = 3$. That means one of the solutions to the equation $5\log x^2 = 3$ is approximately $x = -1.995262$. (It's approximate because the calculator has rounded.)



Now we go through the same process to find the right intersection point. We start by pressing $\boxed{\text{2nd}}$, then $\boxed{\text{TRACE}}$ to get to the calculate menu. After naming the first and second curves and guessing, we end up with the point $x = 1.995263$ and $y = 3$. Here's what it looks like on the calculator.



So the second solution to $5\log x^2 = 3$ is approximately $x = 1.995263$. That makes sense. The graph looks symmetrical, so the positive *x* and negative *x* should be equally far from the origin.

Practice 28

a. Write the domain and range of function: $\log(4x-5)$.

Rewrite each expression below as a single log.

b. $4\log_2 x$

c. $\log_4(5x+10) - \log_4(x+2)$

d. Solve the log equation: $\log_5(x+2) + \log_5(4-x) = 1$.

e. Solve the log equation $3\log(7-x) = 1$ by graphing on a calculator. Round any irrational answers to four decimal places

Answer the word problem below.

f. The park rangers at a wildlife refuge have been keeping track of the bear population in the park. If they've noticed that the annual growth rate is 4% and there are currently 8 bears in the park, how many years will it take for the bear population to reach 20? Round your answer to two decimal places.

Problem Set 28

Tell whether each sentence below is True or False.

1. A log equation is an equation such as $\log(2x-4) = 3$, where the unknown *x* is contained in the log.

2. According to an important rule for logs, $\log M + N = \log M + \log N$.

Write the domain and range of each function below.

3. $\log_2(x+1)$

(a) 4. $\log(3x-9)$

Rewrite each exponential equation into log form.

5. $3^7 = x$

6. $5^x = 4$

Rewrite each log equation into exponential form.

7. $\log x = 6$

8. $\log_x 4 = -2$

Use the change of base formula to find each log below. Estimate your answer to four decimal places.

9. $\log_2 9$

10. $\log_3 5$

11. $\log_{\frac{1}{2}} 14$

Rewrite each expression below as a single log.

12. $\log(x+2)+\log(x-1)$

13. $\log_2(x-4)-\log_2(x+3)$

(b) 14. $2\log x$

(c) 15. $\log_3(4x-8)-\log_3(x-2)$

Solve each log equation below.

16. $\log_2 x = 5$

17. $\log_4(2x+7) = 2$

(d) 18. $\log_3(2x+1)+\log_3(3-x) = 1$

Solve each log equation below by graphing on a calculator. Round any irrational answers to four decimal places

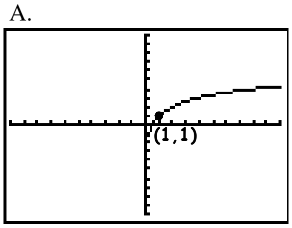
19. $4\log x^3 = 6$

(e) 20. $2\log(5-x) = 1$

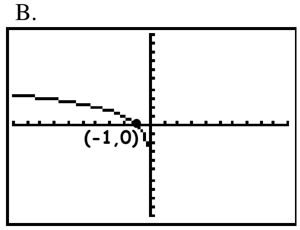
Select the graph for each log function below.

21. $y = -\log_2 x + 3$

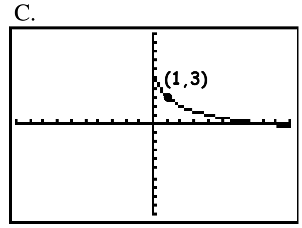
A.



B.



C.



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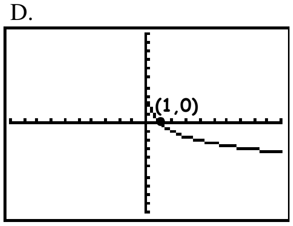

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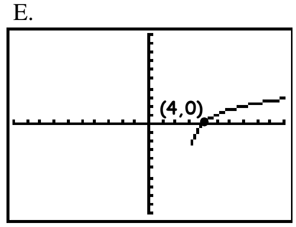
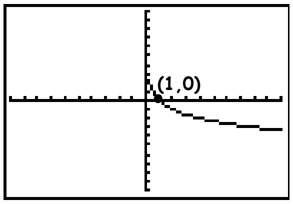


22. $y = 2\log_{\frac{1}{4}} x$

A.



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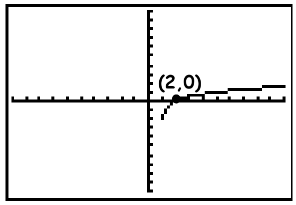

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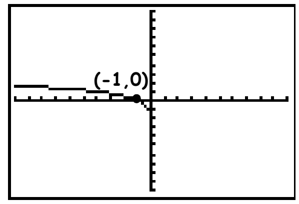

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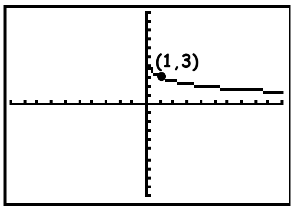
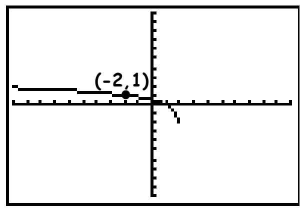


E.



Solve the word problem below.

(f) 23. The owners of a local fishing pond have been keeping track of the trout population in their pond. If they've noticed that the monthly growth rate is 9% and there are currently 36 trout in the pond, how many months will it take for the trout population to reach 60? Round your answer to two decimal places.

Lesson 29—The Base e Exponential Function

So far in this chapter, we've worked with exponential functions that have quite a few different bases: 2, 10, 5, and so on. But believe-it-or-not, the most popular exponential function doesn't have a nice whole number base. It has a base that's equal to the irrational number 2.71828… Since the number is irrational, it can't be written precisely as a decimal or a fraction. The mathematicians actually use the letter *e* to represent 2.71828…, just as they use the Greek letter π to represent 3.14159…

$$e = 2.71828...$$

This new exponential function with base *e* is written as $y = e^x$ or sometimes as $y = \exp(x)$. Since $e = 2.71828...$, the function means $y = (2.71828...)^x$. In case you're wondering, the letter *e* stands for Euler (pronounced "oiler"), because the Swiss mathematician Leonhard Euler was the first person to come up with this way of writing the function $y = e^x$.[7]

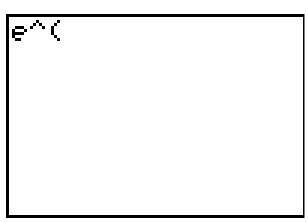
Working with e

The big question, though, is why would people use an exponential function with such a messy base? Why not use something simpler? Because $y = e^x$ is the only function whose rate of change is always exactly equal to the function itself. Remember, we said earlier that all exponential functions have a direct link between the value of the function and the rate of change. When the function doubles, the rate of change doubles. That's what causes the graph of an exponential function to rise faster and faster. But for $y = e^x$, the link between the function's value and the rate of change is perfect. The two are always exactly equal. For example, putting 2 in for *x* in $y = e^x$ gives us approximately $y = 7.3891$.[8]

$$y = e^2 \approx (2.71828...)^2 \approx 7.389$$

That means at $x = 2$, the function $y = e^x$ is growing at a rate of 7.389. The function value and rate of change are always equal. As it turns out, this has great practical value, because it makes $y = e^x$ much easier to work with in higher math, particularly calculus. Since *e* is irrational, the only way to write an exact answer for e^2 is to leave it as e^2. Again it's like π. When you're calculating the circumference of a circle, you usually get an answer like 10π. This can be estimated using 3.14 for π, but 10π is the only way to write that number exactly. The point is that 7.389 is just an estimate for e^2. When solving a practical problem, an estimate is usually just fine. On a math test, the answer should be written as e^2, unless the instructions say specifically to give an estimate.

Almost all scientific calculators have a key for the number *e*. On the TI-83 Plus, the key is actually for e^x. It's in yellow on the far left, three keys up from the bottom. To get an estimate of *e*, just press $\boxed{\text{2nd}}$ and then $\boxed{\text{LN}}$. The calculator then asks you to enter the exponent for *x* in e^x. It looks like this on the screen.



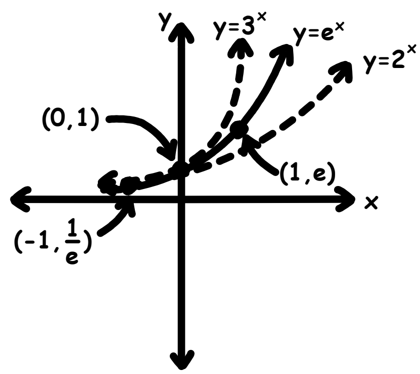
[7] Most irrational numbers, such as $\sqrt{2}$, arise from taking a root. That's why they're written in root form. But since *e* and π don't come about by taking a root, we need letters to represent them precisely.
[8] Remember, the symbol $\approx$ means "approximately equal to."

To get an estimate of *e*, just press 1, since $e^1 = e$. Then press $\boxed{)}$ and $\boxed{\text{ENTER}}$. You should get 2.718281828. That's all the room the calculator has to show digits, but the digits really keep going forever. To calculate e^2, you press $\boxed{\text{2nd}}$, $\boxed{\text{LN}}$, $\boxed{2}$, $\boxed{)}$, then $\boxed{\text{ENTER}}$. That gives 7.389056099, which is again only an estimate.

The Graph of $y = e^x$

Not too surprisingly, the graph of $y = e^x$ looks like all the other exponential functions we've graphed. The curve rises faster and faster as *x* approaches positive infinity. Since $e \approx 2.72$, the graph rises faster than $y = 2^x$, but not quite as fast as $y = 3^x$.

Figure 29.1



You can see that the graph crosses the points $(0,1)$, $(1,e)$, and $(-1,\frac{1}{e})$. That's the same pattern as all other exponential functions. Remember, they all cross the points $(0,1)$, $(1,b)$, and $(-1,\frac{1}{b})$, where *b* is the function's base.

The domain and range of $y = e^x$ are the same as $y = 10^x$. The domain is all real numbers and the range is all positive real numbers.

Domain of $f(x) = e^x = (-\infty, +\infty)$

Range $f(x) = e^x = (0, +\infty)$

It's also possible to transform $y = e^x$ in all the usual ways. The graph can be shifted to the left or right, it can be shifted up or down, and it can also be stretched.

Compound Interest and e

There are lots of practical uses for the function $y = e^x$. One that's fairly easy to understand involves compound interest. Let's say we put $1,000 in a bank that is paying 12% interest per year. The amount of money we'll have after *t* years can be found with the following exponential function.

$$P = 1{,}000(1 + 0.12)^t$$

Remember, the base is 1 plus the growth rate. The growth rate of money in the bank is the interest rate. Written as a decimal, 12% becomes 0.12. Then 0.12 is added to 1 inside the parentheses. To calculate how much money we'll have in the bank after 1 year, we just put 1 in for *t*. The $1,000 will grow to $1,120 (see below left). To calculate how much money we'll have in the bank after 2 years, we put 2 in for *t*. That comes out to $1,254.40 (see below right).

after 1 year	**after 2 years**
$P = 1{,}000(1+0.12)^1$	$P = 1{,}000(1+0.12)^2$
$P = 1{,}000(1.12)$	$P = 1{,}000(1.12)^2$
$P = 1{,}120$	$P = 1{,}254.40$

What if the bank paid interest each month instead of just once a year?

If the bank paid interest each month instead of at the end of each year, the function would change to this.

$$P = 1{,}000(1+\frac{0.12}{12})^t \text{ or } P = 1{,}000(1+0.01)^t$$

Now the growth rate is 1 percent per month, and *t* represents months instead of years. What's interesting is that we make more money when the interest is paid monthly then when it's paid only once per year. One year is 12 months, and if we put 12 in for *t*, we get this.

$$1{,}126.83 = 1{,}000(1+0.01)^{12}$$

See, instead of having $1,120 at the end of the year, we have $1,126.83. The extra $6.83 comes from "interest on interest." When the bank pays us 1% each month, then we can earn more interest on that. The additional interest comes to $6.83 by the end of the year.

If the bank paid interest every day instead of every month, we would earn even more. We would have to adjust the function by dividing 0.12 not by 12 but by 365. And t would represent days instead of months.

$$P = 1{,}000(1+\frac{0.12}{365})^t \text{ or } P = 1{,}000(1+0.000329)^t$$

After one year (365 days), we would have this much money in the bank.

$$1{,}127.47 = 1{,}000(1+0.000329)^{365}$$

Instead of having $1,126.83 at the end of the year, we would have $1,127.47. That's another 64 cents.

If the bank paid interest every hour or every minute, we would earn even more interest. But it wouldn't be very much more. Notice that the increase in interest was $6.83 when the bank went from paying interest yearly to paying monthly. But the increase was only 64 cents when the bank went from paying monthly to daily.

Here's where the number *e* comes in. What if the bank paid interest continuously? Just imagine that the interest money flowed into your bank account every split second as it was being earned. If the bank paid interest continuously, the function would look like this.

$$P = 1{,}000e^{0.12t}$$

The base of the function becomes *e*, which is about 2.71828, remember. And the annual interest rate of 12% (or 0.12 as a decimal) is up in the exponent along with *t*. And *t* now represents years instead of months or days. So to figure out how much we would have in the bank after 1 year when interest is paid continuously, we just put 1 in for *t*.

$$1{,}127.50 \approx 1{,}000e^{0.12(1)}$$

That's $1,127.50, which is slightly more (3 cents!) than we would get if the bank paid daily. The main point is that the base *e* pops up when interest is paid continuously. That's because of the special properties of the function $y = e^x$ that we've already talked about.

To calculate how much money you'll have in the bank after different lengths of time, then, you can use the function $P = P_0(1+\frac{r}{k})^{kt}$ for normal compounding time periods such as years or months or days. The letter P_0 is the amount you put in the bank originally, and *r* is the interest rate the bank pays (written as a decimal). So if the bank is paying 12% per year, you use 0.12. If it's paying 6% per year, you use 0.06. The letter *k* stands for the number of times the bank pays interest in a year. If the bank is paying interest just once a year, you put 1 in for *k*. If it's paying monthly, then you put 12 in for *k*. If the bank is paying interest every day, put 365 in for *k*. Here are a couple of examples.

$$P = P_0(1+\frac{r}{k})^{kt}$$

Interest rate of 10% paid monthly for 7 years: $P = P_0(1+\frac{0.1}{12})^{12(7)}$

Interest rate of 5% paid daily for 3 years: $P = P_0(1+\frac{0.05}{365})^{365(3)}$

To calculate interest paid continuously, you need to use the function $P = P_0e^{rt}$, where once again P_0 is the amount you put in the bank originally, and *r* is the interest rate the bank pays (written as a decimal).

Practice 29

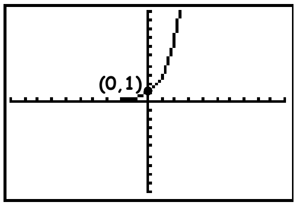
a. Calculate the value of the function $y(3) = 115e^{0.06x}$. Estimate any irrational answer to two decimal places.

b. Rewrite the expression $\frac{\log x}{\log 5}$ as a single log.

c. Solve the log equation $\log x + \log(x+2) = \log 8$.

d. Select the graph for the function $y = e^{-x} - 3$.

A.



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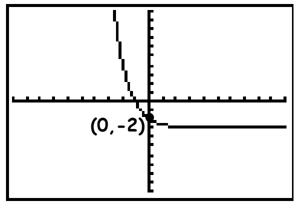
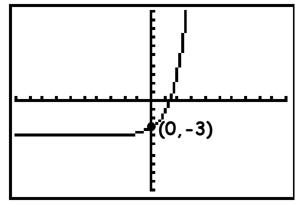
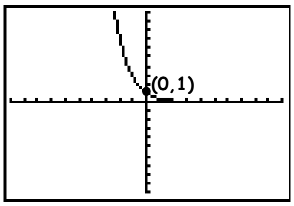


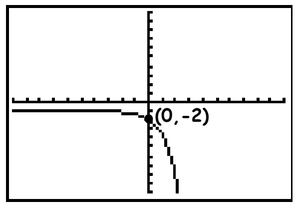
C.



D.



E.



e. We deposit \$1,500 in a bank that is paying 7% interest compounded continuously. Select a function for P representing the amount of money we'll have after t years.

A. $P = 1{,}500(0.07)^t$ B. $P = 1{,}500(1.07)^t$ C. $P = 1{,}500e^{1.07t}$
D. $P = 1{,}500e^{0.07t}$ E. $P = 1.07^t$

f. Solve the word problem below.

Sociologists have found that gossip spreads among a population of people at an exponential rate. Suppose that the function $P = 487(1 - e^{-0.053t})$ represents the number of people (P) in Rumorville (population 487) who have heard the latest gossip t hours after the gossip was first told. How many people will have heard the gossip after 24 hours? Round your answer to the nearest whole number.

Problem Set 29

Tell whether each sentence below is True or False.

1. The domain of the function $y = e^x$ is all positive real numbers and the range is all real numbers.

2. To calculate how much money you'll have in the bank after a certain period of time, you can use the function $P = P_0(1 + \frac{r}{k})^{kt}$, where r is the interest rate and k is the number of times the bank pays interest each year.

Calculate the value of each function below. Estimate any irrational answers to two decimal places.

3. $y(3) = e^x$

4. $y(-2) = e^{x+3}$

5. $y(2) = 1{,}000(1+0.14)^x$

(a) 6. $y(4) = 120e^{0.08x}$

Rewrite each expression below as a single log.

7. $\log_3(x-4) + \log_3(x+4)$

8. $\log(5x-15) - \log(x-3)$

9. $5\log_4 2x$

(b) 10. $\dfrac{\log x}{\log 3}$

Solve each log equation below.

11. $\log_3 x = -4$

12. $\log_2(4-3x) = 5$

(c) 13. $\log x + \log(x+6) = \log 7$

Solve each log equation below by graphing on a calculator. Round any irrational answers to four decimal places

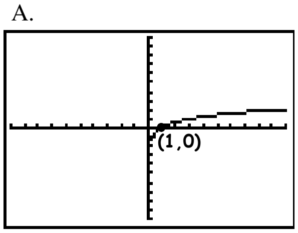
14. $3\log x^4 = 5$

15. $4\log(x-2) = 1$

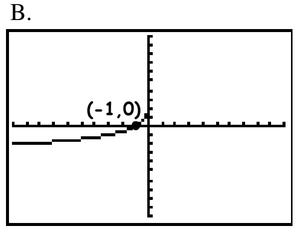
Select the graph for each function below.

16. $y = \log_{\frac{1}{3}}(-x)$

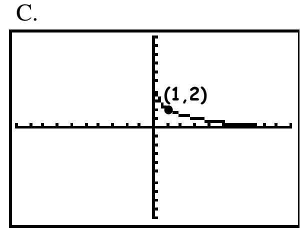
A.



B.



C.



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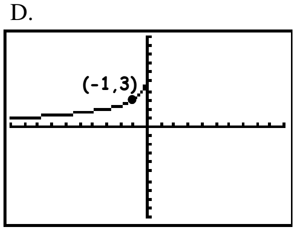
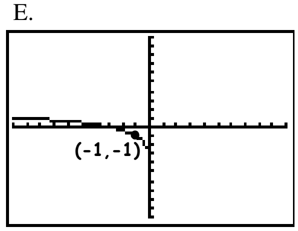
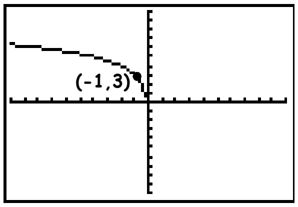


E.

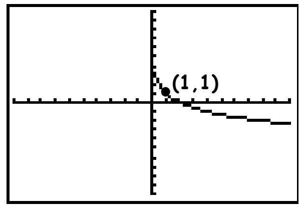


17. $y = \log_2 x - 1$

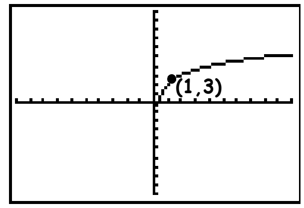
A.



B.



C.



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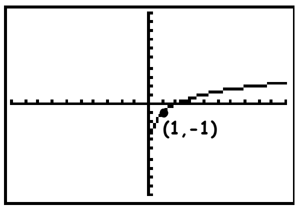
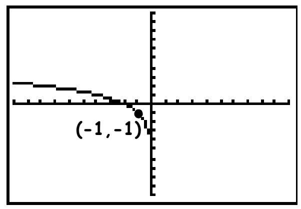
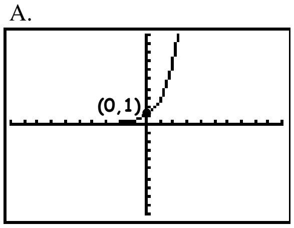


E.

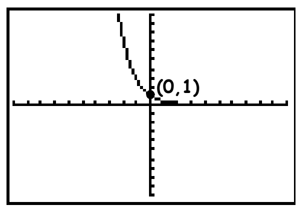


18. $y = e^x$

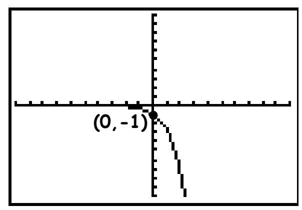
A.



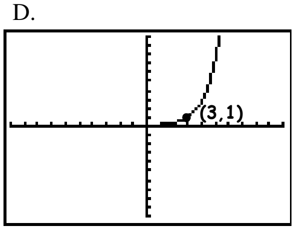
B.



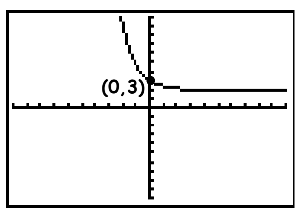
C.



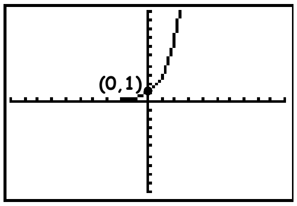
D.



E.



(d) 19. $y = e^{-x} + 2$

A.


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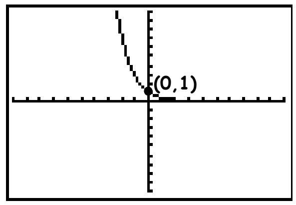

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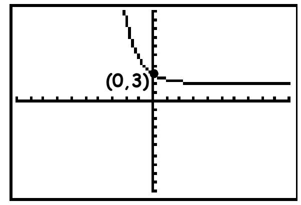

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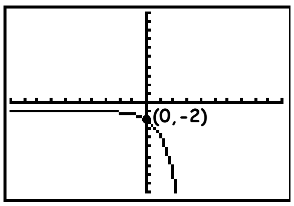

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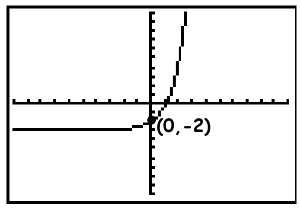


Answer each question below.

20. Phillip deposited \$1,200 in a bank that is paying 9% interest once a year. Select a function for P representing the amount of money he'll have after t years.

A. $P = 1.09^t$ B. $P = 1,200(1.0075)^t$ C. $P = 1,200(1.09)^t$

D. $P = 1,200(0.09)^t$ E. $P = 1,200e^{0.09t}$

21. Edna deposited \$250 in a bank that is paying 6% interest once a month. Select a function for P representing the amount of money she'll have after t months.

A. $P = 250e^{0.06t}$ B. $P = 250(1.005)^t$ C. $P = 1.06^t$

D. $P = 250(1.06)^t$ E. $P = 250(0.06)^t$

(e) 22. Pam deposited \$3,200 in a bank that is paying 8% interest continuously. Select a function for P representing the amount of money she'll have after t years.

A. $P = 3,200(0.08)^t$ B. $P = 3,200(1.08)^t$ C. $P = 3,200e^{1.08t}$

D. $P = 1.08^t$ E. $P = 3,200e^{0.08t}$

Solve the word problem below.

(f) 23. Sociologists have found that gossip spreads among a population of people at an exponential rate. Suppose that the function $P = 532(1 - e^{-0.049t})$ represents the number of people (P) in Slanderburgh (population 532) who have heard the latest gossip t hours after the gossip was first told. How many people will have heard the gossip after 36 hours? Round your answer to the nearest whole number.

Lesson 30—Natural Logarithms

In the last lesson, we learned about base e exponential functions such as $y = e^x$. Remember, e is an irrational number approximately equal to 2.72. It's pretty easy to find a matching value for y in the function $y = e^x$. For instance, if $x = 3$, to find y we just put in 3 for x.

$$y = e^3$$

That gives us e^3, which is the only way to write the answer exactly. It can't be simplified any further. But we can find a decimal estimate for e^3 by using 2.72 for e.

$$y = e^3 \approx (2.72)^3 = 20.09$$

Finding a matching value for x in $y = e^x$ is a bit harder, because it requires solving an exponential equation. For example, if we want to find x when $y = 5$, the first step is to substitute 5 for y.

$$5 = e^x$$

Now we have to get x out of the exponent. We could take the common log (base 10) of both sides.

$$\log 5 = \log e^x$$

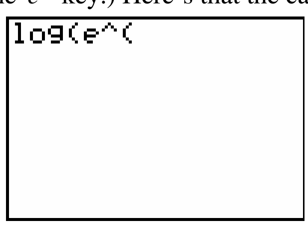
Remember, the power rule says that $\log b^x = x \log b$. Using that rule, we can rewrite the right side like this.

$$\log 5 = x \log e$$

Next, we divide both sides by $\log e$.

$$\frac{\log 5}{\log e} = x$$

Now we need to punch $\log 5$ and $\log e$ into the calculator. To find $\log 5$, we press $\boxed{\text{LOG}}$, then $\boxed{5}$, then $\boxed{)}$, then $\boxed{\text{ENTER}}$. The answer is 0.6990 (rounded to 4 decimal places). To find $\log e$, we can press $\boxed{\text{LOG}}$. Then press $\boxed{\text{2nd}}$ followed by $\boxed{\text{LN}}$. (That's actually the e^x key.) Here's that the calculator shows at this point.



Since we want the log of e, the exponent should be 1 (because $e = e^1$). So we press $\boxed{1}$ then $\boxed{)}$, then $\boxed{\text{ENTER}}$. The answer comes out to 0.4343 (rounded). Putting those numbers in our equation gives us this.

$$\frac{0.6990}{0.4343} = x$$

$$x = 1.6095$$

That's the solution to $5 = e^x$.

Logs in Base e

Solving $5 = e^x$ required several steps, because we took the *common* log of both sides. It was a lot like solving $5 = 2^x$ or $5 = 3^x$. We have to use common logs to solve those equations as well, since calculators don't have base 2 or base 3 log keys. But if the calculator had a log key for base *e*, then solving $5 = e^x$ would be easier. The good news is that since e^x is such an important function, calculators *do* have a special base *e* log key. It's $\boxed{\text{LN}}$. We've pressed $\boxed{\text{LN}}$ before, but it was for e^x, which is in yellow above $\boxed{\text{LN}}$. The $\boxed{\text{LN}}$ key itself is the same thing as "$\log_e$." The LN stands for "natural logarithm" or just "natural log."[9] Instead of writing natural log as "$\log_e$," everybody always uses "$\ln$." It's kind of like writing log instead of $\log_{10}$.

$$\ln \text{ means } \log_e$$

Here's how we can solve $5 = e^x$ faster using the $\boxed{\text{LN}}$ key. We just take the natural log of both sides.

$$\ln 5 = \ln e^x$$

Since ln means $\log_e$, the right side is just $\log_e e^x$, which is the same as *x*.

$$\ln 5 = x$$

The last step is to punch $\ln 5 = x$ into the calculator. We press $\boxed{\text{LN}}$, $\boxed{5}$, $\boxed{)}$, and $\boxed{\text{ENTER}}$ to get 1.6094 (rounded to 4 decimal places). That's extremely close to the answer we got before. The slight difference is due to rounding. But notice how much easier $5 = e^x$ is to solve with natural logs. Since the calculator has a $\boxed{\text{LN}}$ key, we can solve base *e* exponential equations in just a couple of steps. It's no harder than solving a base 10 exponential equation.

We learned several important log properties, such as the product rule, quotient rule, and the power rule. As it turns out, all of those rules also apply to natural logs. Here are they are.

Table 30.1

Basic Rules For Natural Log

1	$\ln e = 1$
2	$\ln 1 = 0$
3	$\ln e^x = x$
4	$e^{\ln x} = x$

Power Rule	$\ln M^x = x$
Product Rule	$\ln M + \ln N = \ln MN$
Quotient Rule	$\ln M - \ln N = \ln \frac{M}{N}$

Changing to Base e

You know that base *e* exponential functions are easy to work with, especially in advanced math. Still, it may seem hard to believe that very many real world exponential functions would work out to have *e* as their base.

[9] Actually, L and N stand for logarithmus naturalis," which is Latin for "natural logarithm."

Wouldn't a base of 2, 5, or 10 be a lot more common? Not really. One reason is that it's possible to change any exponential function to a base of *e*. Let's go through an example.

$$y = 10(2)^{0.4x}$$

This exponential function is currently in base 2. But what if we want to change it to base *e*? All we have to do is find the natural log of 2 by pressing $\boxed{\text{LN}}$, $\boxed{2}$, $\boxed{)}$, and $\boxed{\text{ENTER}}$. That comes out to 0.6931 (to 4 decimal places). So 0.6931 must be the exponent (approximately) that you put on *e* to get 2. Another way of writing that is $e^{0.6931} = 2$. Now all we have to do is substitute $e^{0.6931}$ for 2 in our function.

$$y = 10(e^{0.6931})^{0.4x}$$

According to the exponent rules for algebra, we can multiply the exponents when a power is raised to another power. We can simplify, then, by multiplying 0.6931 and $0.4x$.

$$y = 10(e)^{0.2772x}$$

This is the same function as $y = 10(2)^{0.4x}$, other than slight differences due to rounding. Only $y = 10(e)^{0.2772x}$ now has a base of *e* and is easier to work with. That's how base *e* comes up so often. Basically, we can use base *e* anytime by just rewriting the function into a different form. And since base *e* is easy to work with in higher math, people rewrite functions that way a lot.

The Natural Log Function

The inverse of an exponential function is a log function. So the inverse of $y = 10^x$ is $y = \log x$ (which is base 10). We learned that several lessons ago. As it turns out, $y = e^x$ also has an inverse. It's the natural log function and it's written as $y = \ln x$. The right side is just the log of *x* in base *e* or $\log_e x$. To see why the inverse of $y = e^x$ has to be $y = \ln x$, we can go through the algebra steps. Remember, to find an inverse we just solve for the other variable and then switch the positions of *x* and *y*. To solve $y = e^x$ for *x* the first step is to take the natural log of both sides. (We use the natural log, since $y = e^x$ is in base *e*.)

$$\ln y = \ln e^x$$

On the right, the natural log undoes the power to leave just *x*.

$$\ln y = x$$

Now we switch *x* and *y*.

$$y = \ln x$$

So $y = \ln x$ is the inverse of $y = e^x$. Actually, they're inverses of each other. We can go from $y = \ln x$ back to $y = e^x$ by just rewriting the log function in exponential form. Since $y = \ln x$ means $y = \log_e x$, the function really says that *y* is the exponent that goes on *e* to get *x*. That's $y = e^x$.

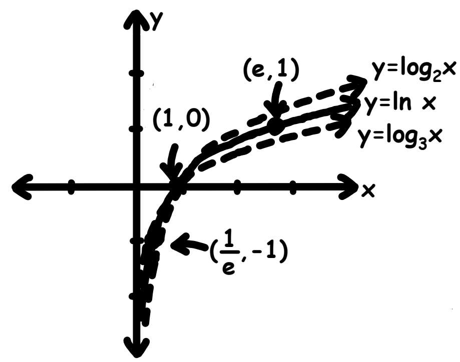
Let's talk about the natural log function, $y = \ln x$. As with other simple log functions (like $y = \log x$ or $y = \log_2 x$), the domain is all positive real numbers. So putting 0 or a negative number in for x in $y = \ln x$ isn't allowed. The range is simpler. It's just all real numbers. Again, that's the same as other simple log functions

Domain of $f(x) = \ln x = (0, +\infty)$

Range $f(x) = \ln x = (-\infty, +\infty)$

Here's the graph of $y = \ln x$, along with the graphs of $y = \log_2 x$ and $y = \log_3 x$.

Figure 30.1



Notice that $y = \ln x$ is between $y = \log_2 x$ and $y = \log_3 x$. That makes sense, because the natural log is base e, and e equals approximately 2.72, which is between the other bases 2 and 3. Since $y = e^x$ and $y = \ln x$ are inverses, they are reflections of each other across the line $y = x$.

The function $y = \ln x$ can also be transformed in different ways. An inside change like $y = \ln(x-1)$ shifts the curve 1 place to the right. An outside change such as $y = \ln x + 5$ shifts the curve up 5. The function $y = \ln x$ can be stretched too through multiplication. One example is $y = 2\ln x$.

Practice 30

a. Calculate the value of the function: $y(4) = \ln(3x)$. Estimate any irrational answer to two decimal places.

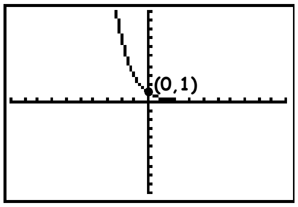
b. Solve the exponential equation $e^{\frac{1}{2}x-3} = 8$ by taking the natural log of both sides. Round any irrational answer to four decimal places.

c. Rewrite the exponential function $y = 6(4)^{0.25x}$ using base e. Round any natural logs to four decimal places.

d. Solve the log equation: $\ln x + \ln(x-3) = \ln 4$.

e. Select the graph for the function: $y = \ln(-x)$.

A.



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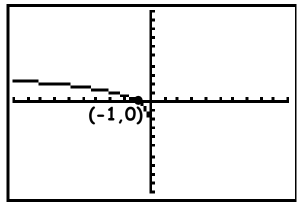

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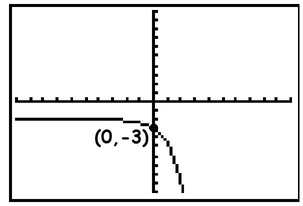

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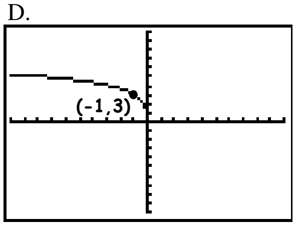

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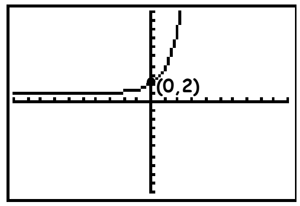


f. Solve the word problem below.

Edgar invested \$800 into a savings account that pays 2.3% annual interest compounded continuously. How much money will he have in the account after 5 years? Round your answer to the nearest cent.

Problem Set 30

Tell whether each sentence below is True or False.

1. A logarithm in base e is called a natural logarithm and is written as “ln.”

2. The fastest way to solve a base e exponential equation (like $5 = e^x$) is to take the natural log of both sides.

Calculate the value of each function below. Estimate any irrational answers to two decimal places.

3. $y(-2) = e^{x+5}$

(a) 4. $y(3) = \ln(2x)$

5. $y(5) = 500e^{0.12x}$

Use the change of base formula to find each log below. Round any irrational answers to four decimal places.

6. $\log_4 8$

7. $\log_7 12$

Rewrite each exponential equation into log form.

8. $2^6 = x$

9. $7^{3x-1} = 5$

Solve each exponential equation below by taking the natural log of both sides. Round any irrational answers to four decimal places

10. $e^{2x} = 25$

11. $e^{1-3x} = 7$

(b) 12. $e^{\frac{1}{2}x-4} = 6$

Rewrite each exponential function below using base *e*. Round any natural logs to four decimal places.

13. $y = 5^x$

14. $y = 2^x + 9$

(c) 15. $y = 8(3)^{0.75x}$

Solve each log equation below. Round any irrational answers to four decimal places.

16. $\log_2(2x+4) = 3$

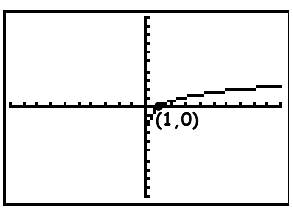
17. $\ln x^3 = 5$

(d) 18. $\ln x + \ln(x-1) = \ln 6$

Select the graph for each function below.

19. $y = -e^{-2x} + 3$

A.



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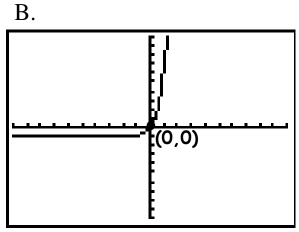

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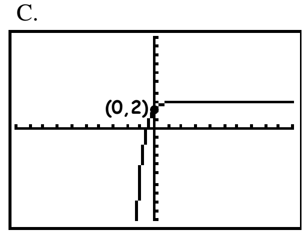

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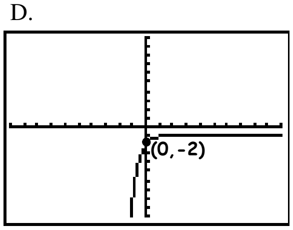

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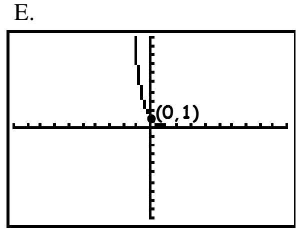
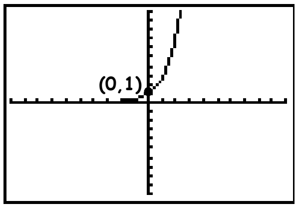


(e) 20. $y = \ln x$

A.



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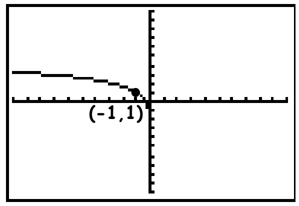

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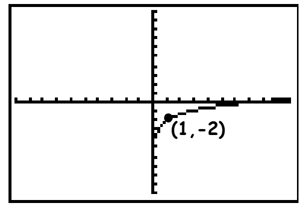

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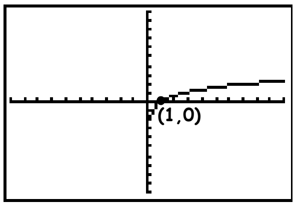

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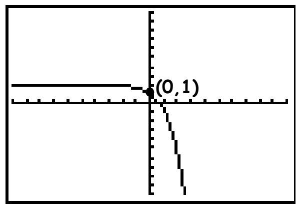

Answer each question below.

21. We deposited \$4,500 in a bank that is paying 9% interest once a month. Select a function for P representing the amount of money we'll have after t months.

A. $P = 4,500e^{0.09t}$ B. $P = 4,500(0.09)^t$ C. $P = 4,500(1.09)^t$
D. $P = 4,500(1.0075)^t$ E. $P = 1.09^t$

22. We deposited \$600 in a bank that is paying 5% interest continuously. Select a function for P representing the amount of money we'll have after t years.

A. $P = 600e^{0.05t}$ B. $P = 600(0.05)^t$ C. $P = 1.05^t$
D. $P = 600(1.05)^t$ E. $P = 600e^{1.05t}$

Solve the word problem below.

(f) 23. Todd invested \$2,000 into a money market account that pays 3% annual interest compounded continuously. How much money will he have in the account after 4 years? Round your answer to the nearest cent.