Teaching Suggestions and/or Solutions to Challengers

Challenger 8 *(p 18)*

1. **(a)** Radius of circular ring $= \frac{1}{2}\sqrt{8^2 + 6^2}$

$= 5$ cm

(b) Total area of shaded parts

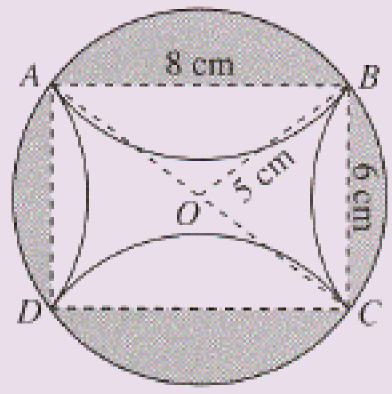
$= [\pi(5^2) - 8 \times 6]$ cm^2

$= (25\pi - 48)$ cm^2

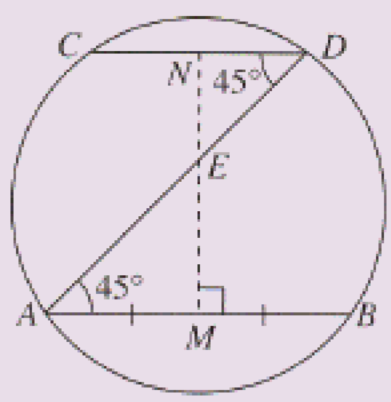
Area bounded by the 4 arcs

$= [8 \times 6 - (25\pi - 48)]$ cm^2

$= (96 - 25\pi)$ cm^2



2.



Draw MN, the perpendicular bisector of AB, cutting AB at M, AC at E and DC at N. So MN passes through the centre of the circle (sym prop of circle).

$MN \perp DC$ ($\because DC // AB$)

Now let $NC = x$ units.

Then $EC = \dfrac{x}{\cos 45°}$

Also $AM = \frac{1}{2}(9\,999)$ units $= 4\,999.5$ units,

$A = \dfrac{4\,999.5}{\cos 45°}$ units

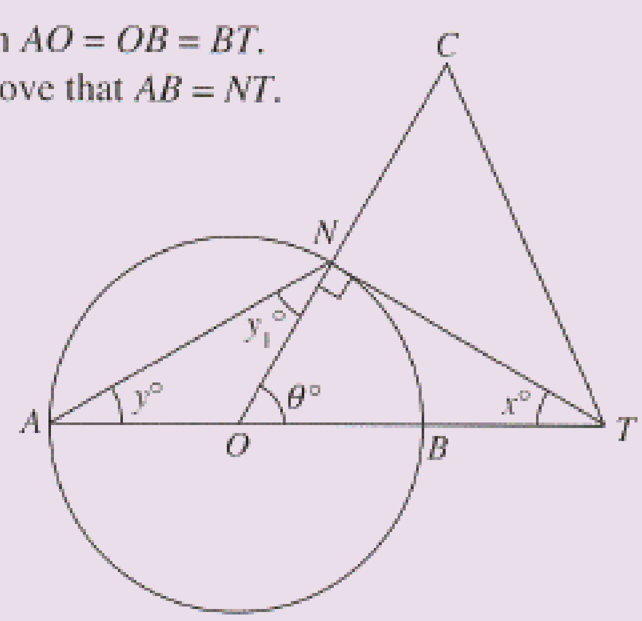
and $AC = AE + EC$

So $7\,777 = \dfrac{4\,999.5}{\cos 45°} + \dfrac{x}{\cos 45°}$

$x = 7\,777 \cos 45° - 4\,999.5 = 499.67$

$\therefore$ CD $\approx$ 999 units

3. Given $AO = OB = BT$.
To prove that $AB = NT$.



Proof:
Produce ON to C such that $ON = NC$.

Since $O\hat{N}T = 90°$ (tangent property of circle),
NT is a line of symmetry of $\triangle TCO$.

$\therefore$ $CT = OT$

Also $OC = OT$ (construction)

$\therefore$ $\triangle OTC$ is equilateral.

$\therefore$ $\theta = 60°$

Now $y° = y_1°$ (isos. $\triangle$)

$\therefore$ $\theta = y° + y_1°$ (ext $\angle$ of $\triangle$)

$= 2y°$

$\therefore$ $y° = 30°$

Also $x° = 180° - 90° - \theta = 30°$

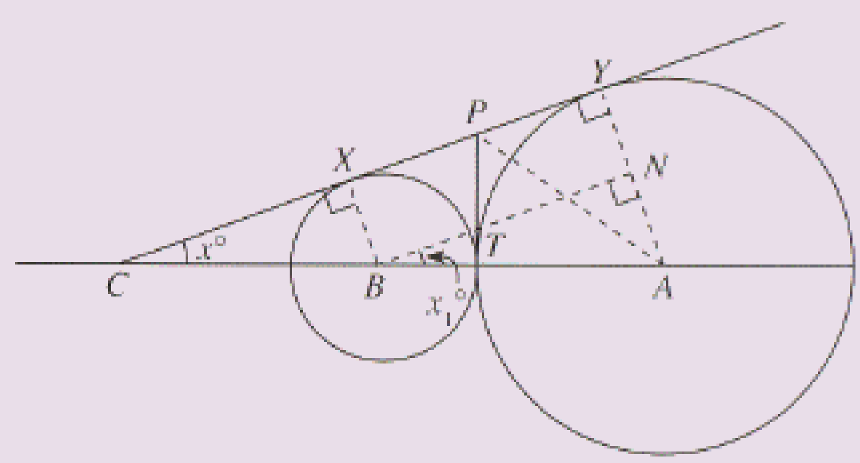
$\therefore$ $y° = x°$

$\therefore$ $AN = NT$ (isos. $\triangle$)

Alternatively if we first prove that $\cos 60° = \frac{1}{2}$, then $\theta° = 60°$ and hence $x° = y° = 30°$ and so on.

Note: After Chapter 9 is taught, you may get your students to do this question again using the property of right angle in a semicircle. Thus, a circle with OT as diameter will pass through N, since $O\hat{N}T = 90°$. It is then easy to prove that $\theta° = 60°$ and hence $x° = y° = 30°$ and so on.

4. Join PA and draw BX, AY and BN as shown.



(a) $\sin x° = \sin x_1°$

$= \dfrac{AN}{BA}$

$= \dfrac{4 - 1}{1 + 4}$

$= \dfrac{3}{5}$

(b) $x° = 36.87°$

$T\hat{A}P = \frac{1}{2}T\hat{A}Y$

$= \frac{1}{2}(90° - x°)$

$= 26.57°$

$PT = TA \tan T\hat{A}P$

$= 4 \tan 26.57°$ cm

$= 2$ cm

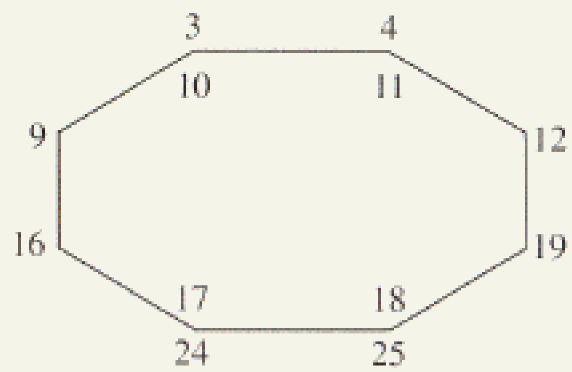
Therefore, the length of PT is 2 cm.

Answers to Investigation

Investigation 3 *(p 95)*

1. **(a)** 112

Note: Each pair of opposite numbers give a sum of 28. So the sum of the numbers on the sides of the octagon is 4 × 28, i.e. 112.



(b) $8(n + 11)$

Note: Observe that the smallest number 3 and the greatest number 25 are related as follows:

$$3 \xrightarrow[+7]{} 10 \xrightarrow[+7]{} 17 \xrightarrow[+7]{} 24 \xrightarrow[+1]{} 25$$

i.e. $3 \xrightarrow{+22} 25$

Likewise the smallest number n and the greatest number N are related as follows:

$$n \xrightarrow{+22} N \quad \text{or } N = n + 22$$

So if the top left number of an octagon is n, the sum of the numbers on its sides is $4(n + N)$, i.e. $8(n + 11)$.

(c) 111

(d) 450 is not divisible by 8.

2. $190; \frac{1}{2}n(n-1)$

Note: Number of angles formed by line 1 with line 2, or 3, or 4, … or 29 is 19.
Number of angles formed by line 2 with line 3, or 4, or 5, … or 20 is 18.
Number of angles formed by line 19 with line 20 is 1.
So total number of angles formed is $(1 + 2 + 3 + \ldots + 19)$, i.e. 190.
If n lines are drawn to meet at O, the number of angles formed is

$$[1 + 2 + 3 + \ldots + (n-1)], \text{ i.e. } \frac{1}{2}n(n-1).$$

3. **(a)**

$1^3 = 1$	1
$1^3 + 2^3 = 9$	$1 + 2 = 3$
$1^3 + 2^3 + 3^3 =$ (36)	$1 + 2 + 3 =$ (6)
$1^3 + 2^3 + 3^3 + 4^3 =$ (100)	$1 + 2 + 3 + 4 =$ (10)

(b) $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots n)^3$

(c) $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

(d) True

Note: $S^n = 1^3 + 2^3 + 3^3 + \ldots + n^3;$

$$S_{n-1} = 1^3 + 2^3 + 3^3 + \ldots + (n-1)$$

$$n^3 = S_n - S_{n-1} = \left[\frac{n(n+1)}{2}\right]^2 - \left[\frac{(n-1)(n)}{2}\right]^2$$

where $\frac{n(n+1)}{2}$ and $\frac{(n-1)(n)}{2}$ are whole numbers.

(e) $5\,050^2 - 4\,950^2$

Note: $(100)^3 = \left(\frac{100(101)}{2}\right)^2 - \left(\frac{99(100)}{2}\right)^2$

So $1\,000\,000 = 5\,050^2 - 4\,950^2$.

4. **(a)**

A cube made of $n \times n \times n$ unit cubes	
n	**Number of cubes of various sizes**
1	No. of 1 × 1 × 1 cubes = (1)
2	No. of 1 × 1 × 1 cubes = (8)
	No. of 2 × 2 × 2 cubes = (1)
	Total = (9)
3	No. of 1 × 1 × 1 cubes = (27)
	No. of 2 × 2 × 2 cubes = (8)
	No. of 3 × 3 × 3 cubes = (1)
	Total = (36)
4	No. of 1 × 1 × 1 cubes = (64)
	No. of 2 × 2 × 2 cubes = (27)
	No. of 3 × 3 × 3 cubes = (8)
	No. of 4 × 4 × 4 cubes = (1)
	Total = (100)
5	No. of 1 × 1 × 1 cubes = (125)
	No. of 2 × 2 × 2 cubes = (64)
	No. of 3 × 3 × 3 cubes = (27)
	No. of 4 × 4 × 4 cubes = (8)
	No. of 5 × 5 × 5 cubes = (1)
	Total = (225)