10.1 THE IDEA OF A SET

A **set** is a collection of things. In our everyday speech, some sets have special names. For example, we talk of a herd of cattle, a fleet of battleships or a team of soccer players. In mathematics, we simply say a set of cattle, a set of battleships, and so on.

MATHSTORY

Georg Cantor, 1845–1918, a German mathematician, pursued a career in mathematics and worked on set theory and transfinite arithmetic. However, his work gained little recognition during his lifetime.

The things that make up a set are called **elements** or **members** of the set. For example, the letters a, e, i, o and u are the elements of the set of vowels of the English alphabet. We may wish to denote this set by a capital letter A, and write:

$$A = \{i, e, a, u, o\}$$

Notice that the order in which we write down the elements is immaterial.

The curly brackets have the meaning 'the set whose elements are'. The letter u is an element of set A, and we write:

$$u \in A$$

The letter b is a consonant and it is not an element of set A. We write:

$$b \notin A$$

We use **n(A)** to denote the number of elements in set A. Since there are 5 elements in set A, we write $n(A) = 5$.

How many elements are there in each of the following sets?

$$P = \{1, 2, 3, 4, \ldots\}$$
$$Q = \{1, 2, 3, 4\}$$
$$R = \{1, 2, 3, 4, \ldots, 1\,000\,000\}$$

P is the set of natural numbers. There are infinitely many elements in this set. This is an **infinite set**.

Q is the set of natural numbers less than 5. This is a **finite set**. The number of elements in Q is 4, and we write $n(Q) = 4$.

R is the set of the first million natural numbers and is a finite set. This set has 1 000 000 elements. Thus $n(R) = 1\,000\,000$.

A set is a finite set if the number of elements in the set is zero or a natural number. An infinite set is a set that is not finite.

Equivalent Sets and Equal Sets

Examples

(a) A is the set of letters in the word '*number*'.
B is the set of letters in the word '*figure*'.
Notice that every element of A can be matched with exactly one element of B like this:

$$\begin{array}{l} A = \{n,\ u,\ m,\ b,\ e,\ r\} \\ \quad\ \ \updownarrow\ \ \updownarrow\ \ \updownarrow\ \ \updownarrow\ \ \updownarrow\ \ \updownarrow \\ B = \{f,\ i,\ g,\ u,\ r,\ e\} \end{array}$$

We say that A and B are **equivalent sets**. Notice also that $n(A) = n(B)$.

11.2 USING VENN DIAGRAMS TO SOLVE PROBLEMS

Worked Example 4

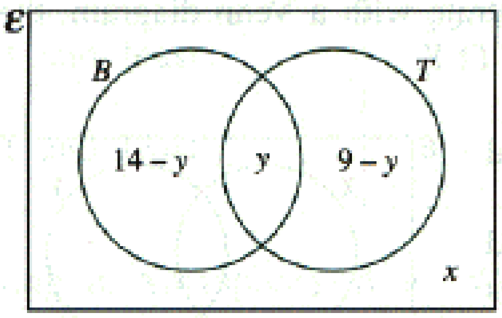
In a group of 20 students, 14 play badminton, 9 play table tennis and x do not play either of the games.

(a) If $x = 0$, find the number of students who play both games.

(b) If $x \neq 0$, find the greatest possible value of x.

Solution:

(a)



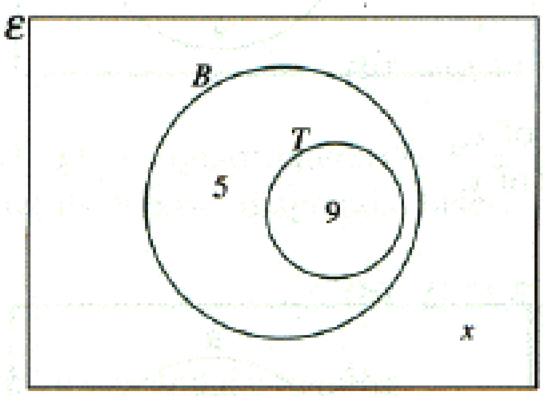
Let B = {students who play badminton},
T = {students who play table tennis}
and $y = n(B \cap T)$.

If $x = 0$, we have

$$14 - y + y + 9 - y = 20$$
$$y = 3$$

Thus the number of students who play both games is 3.

(b)



x has the greatest value when $n(B \cup T)$ is smallest.

$$x + 5 + 9 = 20$$
$$x = 6$$

Thus the greatest value of x is 6.

14.5 PERCENTILES, QUARTILES AND THE INTERQUARTILE RANGE

Percentiles and Quartiles

Example

The cumulative frequency table for data taken from a mathematics test of 50 students is shown below.

Marks	Number of students who obtained this mark or less	Percentage cumulative frequency
9	1	2
19	3	6
29	6	12
39	10	20
49	19	38
59	34	68
69	41	82
79	45	90
89	48	96
99	50	100

Notice that a column headed 'Percentage cumulative frequency' is included. Each cumulative frequency is worked out as a percentage of the total number of students taking the test and recorded in the last column. For example, $\frac{1}{50} \times 100\% = 2\%$; $\frac{3}{50} \times 100\% = 6\%$ and so on. We represent this information by a cumulative frequency curve as shown on the next page.

Study the cumulative frequency curve. Corresponding to 'half the population', that is, 50%, is the mark of 51. This is the median (Q_2) and is also known as the **50th percentile**. This means that 50% of the students obtained 51 marks or less.

The **25th percentile** (Q_1), which is also known as the **lower quartile** corresponds to the mark of 43. It tells us that 25% of the pupils obtained 43 marks or less, that is, three quarters of the students obtained above this mark (43) and a quarter not above it.