1.1 POSITIVE INTEGRAL INDICES

We know that 7^5 means $7 \times 7 \times 7 \times 7 \times 7$. The number 7 is called the **base** and the number 5 is called the **index** or **exponent**.

7^5 is read as '*the 5th power of 7*' or '*7 to the power 5*'. Similarly, 10^4 is read as '*the 4th power of 10*' or '*10 to the power 4*'.

Numbers in index notation with the same base 7 such as 7^5, 7^9 and 7^{12} are referred to as **powers** of 7. Similarly, 10^4, 10^8 and 10^{11} are powers of 10.

7^5
base
index or exponent

The exponent tells you how many times a number, or base, is used as a factor.

MATHSTORY

The French mathematician René Descartes (1596–1650) introduced the use of Hindu-Arabic numerals as exponents on a given base.

Examples

(a) Consider $7^3 \times 7^2$.
We have $7^3 \times 7^2 = (7 \times 7 \times 7) \times (7 \times 7)$
$= 7^5$

Notice that 7^5 can be written as 7^{3+2}.
So we have $7^3 \times 7^2 = 7^{3+2}$.
In general, we have the first Law of Indices:

$$a^m \times a^n = a^{m+n}$$

(b) Consider $\frac{3^6}{3^4}$.

We have $\frac{3^6}{3^4} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3}$
$= 3 \times 3$
$= 3^2$

Notice that 3^2 can be written as 3^{6-4}.

So we have $\frac{3^6}{3^4} = 3^{6-4}$.

In general, we have the second Law of Indices:

$$\frac{a^m}{a^n} = a^{m-n}$$

Note: We assume that $m > n$ since negative indices are not dealt with in this section.

(c) Consider $(5^2)^3$.
We have $(5^2)^3 = 5^2 \times 5^2 \times 5^2$
$= (5 \times 5) \times (5 \times 5) \times (5 \times 5)$
$= 5^6$

Notice that 5^6 can be written as $5^{2 \times 3}$.
So we have $(5^2)^3 = 5^{2 \times 3}$.

MISCELLANEOUS EXERCISE 1

(answers on p. 431)

1. Without using a calculator, simplify $(-2)^3 + (-2)^{-4} + \left(-\frac{1}{2}\right)^{-2} - \left(-\frac{1}{2}\right)^2$.

2. Without using a calculator, simplify $(7 \times 17^{-1} - 7^{-1} \times 17)(7^{-1} - 17^{-1})^{-1}$.

3. **(a)** Find, without using a calculator, the last digit (i.e. the digit at the 'ones' place) of each of the following.
(i) 19^2 **(ii)** 19^3 **(iii)** 19^4 **(iv)** 19^5 **(v)** 19^{19}
(b) Find the last digit of 1998^{1998}.

4. Find the value of $\frac{x}{y}$ if $3x^2 - 4xy + y^2 = 0$.

5. Solve $\left(x + \frac{1}{x}\right)^2 - \left(x + \frac{1}{x}\right) - 6 = 0$. $\left(\textit{Hint:}\ \text{Let } y = x + \frac{1}{x}.\right)$

6. If $(2a)^{2b} = (a^b)(x^b)$, find x in terms of a.

7. If a, b and c are prime numbers such that $a = b^2 - c^2$, find a.

8. When Abu's age was x years, his father's age was 3 years more than twice his age. Abu's age is now y years. Find an expression for his father's present age.

9. **(a)** Given that $a + b = 3$ and $a - b = -7$, find the value of $a^2 - b^2$.
(b) Given that $x^2 + y^2 = 25$ and $xy = 12$, find the value of
(i) $(x + y)^2$,
(ii) $(x - y)^2$.

10. **(a)** If the product of two possible integers is a prime number, what must be the value of the smaller integer?
(b) Given that x and y are positive integers, solve
(i) $(x + y)(x - 3y) = 17$,
(ii) $x^2 - 4y^2 = 13$.

11. If $2x - 3y + 1 = 0$, express $3x - y$
(a) in terms of x,
(b) in terms of y.

6.1 SIMPLE ELIMINATION METHOD

We have seen that simultaneous linear equations can be solved by using graphs. We shall now learn how they can be solved by using an algebraic method known as the elimination method.

Examples

(a) Consider these simultaneous linear equations.

$$6x + 4y = 24 \quad \text{.......................} \quad (1)$$
$$7x - 4y = 2 \quad \text{.......................} \quad (2)$$

As the terms $+4y$ and $-4y$ are found in equation (1) and equation (2), the variable y can be eliminated easily by forming a new equation as follows:

(1) + (2): $$(6x + 4y) + (7x - 4y) = 24 + 2 \quad \text{.....................} \quad (3)$$
$$13x = 26$$
$$\therefore x = 2$$

Notice that the left-hand side of equation (3) is the sum of the left-hand sides of equations (1) and (2). The right-hand side of equation (3) is similarly obtained.

To find the value of y, we take x to be 2 in either equation (1) or equation (2). Thus if we replace x by 2 in equation (1), we have

$$(6 \times 2) + 4y = 24$$
$$12 + 4y = 24$$
$$4y = 24 - 12$$
$$4y = 12$$
$$\therefore y = \frac{12}{4}$$
$$= 3$$

$\therefore$ the solutions are $x = 2$ and $y = 3$.

Check: $7x - 4y = 7 \times 2 - 4 \times 3 = 2$, as in equation (2).

Note: We use equation (2) for the check because the value of y was found by putting $x = 2$ in equation (1).

(b) Consider these simultaneous equations.

$$2x + y = 10 \quad \text{.......................} \quad (1)$$
$$2x - 3y = 2 \quad \text{.......................} \quad (2)$$